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STUDIES OF MULTI-VARIABLE MANUAL CONTROL SYSTEMS:

TWO AXIS COMPENSATORY SYSTEMS WITH COMPATIBLE INTEGRATED DISPLAY AND CONTROL

by William H. Levison and Jerome I. Elkind

Prepared by
BOLT BERANEK AND NEWMAN, INC.
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ABSTRACT

Experiments were conducted to determine what modifications to the current models of the human controller of single-variable systems are necessary for them to be good representations of the controller in two-variable situations. These experiments were performed with a single compensatory display and a single two-axis control. Two descriptors of performance were obtained for each axis: (1) the normalized mean squared error, and (2) the describing function. Of prime interest was the extent to which performance on a given axis was modified by the requirement of simultaneously tracking a second axis. Three two-axis control situations were investigated: (1) homogeneous control situation, in which the input power spectra and controlled elements were identical on X and Y, (2) heterogeneous inputs, in which the input bandwiths were different but the controlled elements identical, and (3) heterogeneous dynamics, in which the controlled-element dynamics were different but the input bandwidths identical.

Two-axis performance degradation was small when the tracking conditions were homogeneous and when the inputs (but not the dynamics) were heterogeneous. Large and significant performance differences were seen when the dynamics were heterogeneous. In this situation the increase in normalized mean squared error ranged from 15% to 125%, depending on the subject and the axis under consideration. In addition, there were important changes in the controller's equalization.

Three factors that affect human controller characteristics in two-axis control situations are identified. These are: (1) visual-motor interaction, (2) differential allocation of attention, and (3) non-homogeneity of required equalization when the controlled-element dynamics are non-homogeneous.

A simple model has been developed for predicting visual-motor interference effects. Models for the prediction of attention and equalization effects have not yet been developed. Single-axis describing function models for the human controller should be modified to include the effects of these factors in order to obtain accurate predictions of human controller characteristics in two-axis situations and probably also in higher-dimensional control situations.

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STUDIES OF MULTI-VARIABLE MANUAL CONTROL SYSTEMS:

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I. INTRODUCTION

Most of the laboratory research on manual control systems directed toward the development of mathematical models for the human controller has dealt with single-loop, single-axis systems, (Refs. 1-3). Mathematical models for the human controller that provide accurate predictions of his behavior over a wide range of single-axis control situations have resulted from these studies. The recent report by McRuer, Graham, Krendel and Reisner (Ref. 3) is the most comprehensive and advanced contribution to the modelling of such single variable systems yet to appear. However, most systems of practical importance are multi-variable systems and, therefore, it is important to develop models for the human controller that are applicable to situations in which he is controlling several variables simultaneously. A good strategy for developing such models is to build upon the existing single-variable models. Since several investigators (Refs. 4-8) have demonstrated that human controller behavior in a two-axis control situation is different from that in a single-axis system, it is evident to make them applicable to multi-variable control situations. The nature and extent of the modifications that we required have not yet been determined.

The research discussed in this report was the first phase of a continuing theoretical and experimental study of multi-variable manual control systems. The purpose of this research was to investigate in detail the extent to which the current single-variable models for the human controller apply to two-axis control situations and to determine what modifications to these models are necessary for them to be good representations of human controller behavior in two-axis situations.

In the next Section of this report, Section II, we review briefly the status of single-variable describing function models for the human controller. In addition we review the pertinent literature on multi-variable human controller behavior. With this background as a basis we present several hypotheses relating to possible differences between single- and two-variable control behavior. Section III is a description of the experimental program. It contains a summary of the experiments performed, a description of the apparatus used, and a description of the procedure followed during the experiments. The analysis methods and performance measures that we used to analyze and present the experimental results are described in Section IV. Comparisons are made between our results and those obtained by McRuer et al (Ref. 3). The experimental results are presented in Section V. The results are discussed in Section VI, and modifications of the single-variable human controller model are suggested to provide a good representation of the human controller in a two-axis compensatory control situation with compatible integrated control and display.

II. BACKGROUND

A. THE HUMAN CONTROLLER IN SINGLE-VARIABLE MANUAL CONTROL SYSTEMS

1. Describing Function Representations

In Fig. 1 is a block diagram of a flight control system. The pilot views a display and responds to the information displayed on it by moving the control device. The control device provides signals to the vehicle (controlled element) whose dynamics are represented by the transfer function C(s). Information about the response of the vehicle is processed and fed back to the display.

Most of the describing function studies have been performed with a compensatory display, an example of which is in Fig. 2. The displacement of the single indicator, the dot, is proportional to the tracking error. The human controller's task is to move the control device so as to correct or to compensate for this error. If the dynamics of the control device are negligible compared to those of the hand or arm, if the display is compensatory, and if the displayed error is the only stimulus to the operator, the dynamic characteristics of the system of Fig. 1 can be represented by the simpler block diagram of Fig. 3. The dynamic characteristics of the human pilot, which are non-linear, noisy, and time-varying, can be represented by a quasi-linear operator H(s) (the describing function) and a remnant noise $n_h(t)$, added to the output of H(s).

2. Mathematical Models

The most comprehensive discussion of single-variable models of the human controller appears in McRuer et al (Ref. 3). They offer models of varying degrees of complexity to describe human control behavior in a wide variety of tracking tasks.

The simplest model states that the human controller adjusts his characteristics so that the combined pilot-vehicle describing function will have a gain that decreases at a rate of 20 db/decade in the region of gain crossover. Thus

$$HC(j\omega) = \frac{\omega_c \, e^{-j\omega \tau_e}}{j\omega} \tag{1}$$

where ω_c is the gain crossover frequency and τ_e is an effective time delay which includes neural conduction time, central processing time, and the effects of high-frequency poles. This model is intended to be valid only in the region of the gain crossover frequency (i.e., the frequency at which HC = 0 db). Nevertheless, it is a useful model for predicting system performance, since a large fraction of the spectrum of the tracking error is often concentrated in a narrow frequency range that encompasses the gain-crossover frequency.

The gain crossover model implies that the human controller adapts his dynamic behavior to that of the controlled-element so that HC remains approximately constant, at least in the region of crossover.

A simple describing-function model that illustrates explicitly the adaptive capabilities of the human controller is:

$$H(s) = K_h \frac{(T_L s + 1)}{(T_I s + 1)} \cdot \frac{e^{-\tau s}}{(T_N s + 1)} \qquad (2)$$

The lead-lag term $(T_L s+1)/(T_I s+1)$ is an equalizer which together with the gain K_h is adjusted by the human controller to achieve a good system performance. The delay τ and the lag $1/(T_N s+1)$ approximate the dynamic characteristics of the neuro-muscular system. Experimentally-obtained estimates of τ are in the neighborhood of 0.09 second. When tracking with controlled-element dynamics of K (which is one of the controlled elements used in our experiments), the controller generates a small lead time constant of about .11 sec, which has the effect of cancelling the effects of the neuro-muscular lag. He also employs a lag time constant and a gain K_h such that the gain crossover frequency is in the neighborhood of 8 rad/sec. When tracking with K/s^2 dynamics (the second dynamics used in our experiments) the controller generates a lead time constant of about 5 sec, has a neuro-muscular time constant T_N of about .11 sec, and essentially no equalizer lag T_I. He adjusts the gain K_h to achieve a gain crossover frequency of about 4 rad/sec.

B. HUMAN PERFORMANCE OF MULTI-VARIABLE TASKS

1. General Multi-Variable Tasks

In order to construct a multi-variable model of the human controller, we must determine to what extent the human is able to process more than one channel of information simultaneously. If he is able to operate on only one variable at a time, then a multi-variable model must include a switching or scanning mechanism to account for sequential processing. On the other hand, if he is able to operate on several task variables simultaneously, we may be able to construct a multi-variable model that is a simple parallel combination of single-variable models.

Multi-variable situations which require sampling will generally produce a degradation in the performance of one or more of the component tasks. For example, overt visual sampling is required when pertinent information is presented on widely separated displays. Motor sampling is necessary if the human is required to manipulate more than one control with the same hand.

Of fundamental importance to the entire question of multi-variable manual control systems is whether or not there will be performance degradations when overt visual and motor sampling are not required. Some multi-variable displays which do not require overt sampling are (a) two or more closely spaced lights, all of which can be monitored simultaneously, (b) a two-dimensional oscillographic display, and (c) a combined single-visual and a single-variable auditory display. The subject may execute a multi-variable response by (a) depressing a number of keys, each with a different finger, (b) manipulating a two- or three-dimensional control with one hand, or (c) manipulating two controls, one with each hand, for example. The individual-task performance may be degraded in this type of multi-variable situation because of covert sampling or some other form of interference.

There is evidence that performance is degraded as the number of task variables increases, even though overt sampling is not required. For example, Brainerd et al (Ref. 9) found that reaction time increased on the average as the number of stimulus-response channels increased. Since in his experiments all the visual stimuli could be monitored simultaneously and all motor responses executed simultaneously, no overt sampling was required. There was, therefore, interference at the central processing level as the number of tasks was increased. Similarly, Kristofferson (Ref. 10) found that reaction time was greater when the subject was presented information on two sensory modalities than when information was presented on only one. He accounted for the task interference by a central switching of attention between the two sense modalities in the two-variable task.

2. Multi-Variable Tracking

Todosiev et al (Ref. 4) recently completed a series of two-dimensional tracking experiments in which one of the objectives was to compare single-axis to two-axis performance. The controlled-element dynamics were of the form $K/s(Ts+1)$ and were the same for both axes. The subjects were presented with integrated control and display configurations.

The authors found that the lead time constant of the human controller's describing function was significantly greater in the two-axis situation than in the single-axis situation for some of the tracking tasks studied. On the other hand, they did not find a significant difference between the two-axis mean squared error performance and the single-axis performance. Since the authors did not present the means and variances of the single- and the two-axis error scores, we are unable to judge the sensitivity of their experiments.

Verdi et al (Ref. 5) investigated single-axis and two-axis tracking behavior with controlled dynamics of acceleration. Quickening was added to one or both of the axes in some of the experiments. When there was no quickening, the authors found a one-axis, two-axis difference in mean squared tracking error of 65% which they claim was not statistically significant. In addition, they found no significant one-axis, two-axis differences in mean squared error when the dynamics were quickened in both axes or when the dynamics wer quickened in only one axis. They found a significant one-axis, two-axis change in the describing function only when one axis was quickened and the other axis was unquickened.

Additional studies have been conducted to investigate various configurations of multi-variable control. In these studies, human performance was described in terms of an appropriate error measure only. The changes in the describing function resulting from changes in the multi-variable control situation were not investigated.

Duey and Chernikoff (Ref. 6) compared single- and two-axis performance when the controlled-element dynamics were either pure acceleration or quickened acceleration. When the dynamics in both axes were pure acceleration, the authors found that two-axis tracking caused a significant increase of 30% in integrated absolute error. When the dynamics in both axes were quickened acceleration, they found no significant difference between the single-axis and two-axis performances. When the dynamics were mixed--pure acceleration on one axis and quickened acceleration on the other--performance in both axes was degraded by two-axis tracking.

Chernikoff et al (Ref. 7) investigated two-axis tracking performance using various pairs of controlled-element dynamics. The controlled dynamics were either K, K/s, or K/s^2. The subjects were presented with integrated control and display configurations. For a given set of dynamics, they found that performance was best when the dynamics in the two axes were the same. The performance degradation increased as the difference between the X and Y dynamics increased. The authors did not compare single-axis performance to two-axis performance with identical dynamics. It is unlikely that two-axis performance was superior to single-axis performance under any circumstances. We can reasonably conclude from their experiments, therefore, that performance on a given axis of a mixed-dynamics, two-axis, tracking task was inferior to performance on that axis when tracked alone.

In a more recent study, Chrenikoff and LeMay (Ref. 8) investigated two-axis tracking performance using various control-display configurations as well as various paris of controlled-element dynamics. The dynamics were either K or K/s^2. The subject was presented with a single two-variable display or two single-variable displays, and with a single two-axis control or two single-axis controls, one for each hand. The presentation of two displays required visual sampling; the operation of two controls, however, did not require overt motor sampling.

When the X and Y dynamics were the same, performance was best with a single-display, single-control configuration. As expected, performance was degraded consistently by the replacement of the single display by the two separated displays. Performance was degraded to a lesser extent by the use of two controls, presumably because the subject could not make an integrated motor response as well with two controls as with a single control. Of greater interest are the Chrenikoff and LeMay results with different dynamics on the two axes. Significantly better performance was obtained with separated controls than with a single two-axis control in this situation whereas the type of display had little effect.

A few models of the two-axis human controller are suggested by these results. One is that the two different types of control responses demanded when tracking K and K/s^2 dynamics simultaneously produce motor interference when only a single control is used. That is, motions intended for one axis may have a component on the other, with adverse effects. A separation of controls requires only one type of control response from each hand and should reduce motor interference. When the dynamics are the same, the control responses are of the same form in the two axes. The effects of motor interference should therefore be less, and little benefit should accrue from a separation of the controls.

Another model, not mutually exclusive with the motor-interference model, is that the complexity of the mixed-dynamics task is so great that the subject is forced to process the two axes separately. Separated controls allows him to maintain channel separation throughout. On the other hand, the two axes are processed in parallel when the controlled dynamics are identical. A single control is more efficient in this task. Hence, the multi-axis task is reduced in effect to a single-axis situation, because it allows the human to behave as a single-channel device throughout.

3. Pre-experiment Model of a Two-variable Human Controller

Figure 4 is a block diagram of a two-axis manual control system which contains integrated control and display configurations. The human controller monitors an integrated two-dimensional display of the error and responds by manipulating a control device in two orthogonal dimensions. Each of the two orthogonal components of effector output is fed to a controlled element.

The human controller is represented as a modified parallel processor in the block diagram of Fig. 5. The sensory processor (visual system) monitors the error display and resolves the error into X and Y components. The central processor determines the appropriate strategies (equalizer characteristics) for the two axes. Commands are sent to the motor (neuro-muscular) systems corresponding to each axis of control. The X and Y components of hand motion are summed vectorially to yield a two-dimensional response.

If the human is truly a parallel controller, there will be no interaction between signals processed on the two axes, and the performance on one axis will be unaffected by the presence or absence of a simultaneous task on the other. The arrows between corresponding X- and Y-axis functions indicate the possibility of one or more sources of interference, or interaction, between axes. Peripheral sampling, either visual or motor, has been eliminated by consideration of an integrated two-dimensional display and control configuration. Other potential sources of interference are tabulated below according to the signal-processing level at which they might occur.

a. Visual System

Perceptual interference. The presence of errors on one axis may degrade the perception of error and error velocity in the other axis.

b. Central Processing

Information processing limitation. Each axis by itself may require the human to process information at a rate that is greater than half of his capacity. Two such axes simultaneously tracked will demand a processing rate beyond his capability, with consequent performance degradation in one or both axes.

Single-channel behavior. The central processor may be able to handle only one channel at a time and will be forced to switch between axes. Performance degradations will result from the effective time delays added to the describing functions on each axis.

Generation of different equalizations. When the controlled-element dynamics are not identical, the human controller will be required to generate different describing functions on X and Y. The central processor may react to the complexity of the tasks by (1) switching between channels, as discussed above, or (2) failing to maintain the difference between the X- and Y-axis describing functions. That is, the two describing functions will be more similar to one another when two axes are tracked simultaneously than when they are tracked individually. Therefore, the strategy on one or both axes will be non-optimal in the two-axis situation, and performance will be degraded.

Variability. The human controller may be less able to maintain close control over his characteristics in the more complex two-variable tracking task than in the single-variable task. If the variability of the describing function increases, the controller may have to increase his phase margin (and consequently tolerate a degradation in system performance).

Attention. If the component tasks of a two-variable task are of unequal difficulty, the subject may assign a higher cost function to the more difficult task and therefore concentrate on it to the partial exclusion of the easier task when the two axes are tracked simultaneously.

c. Motor System

Motor interference. Motions intended for one axis may inadvertently produce components of motion in the other. This type of interference may occur because the subject does not make a control motion in precisely the direction intended.

Motor interference will generally be uncorrelated with the input signal on the axis on which interference occurs and will therefore increase the rms tracking errors.

III. EXPERIMENTAL PROGRAM

A. OBJECTIVES OF EXPERIMENTS

The purpose of our experimental program was to determine the extent to which the human controller's behavior in two-axis control situations differs from that in single-axis sutuations, and to attempt to identify the source and type of interference that is responsible for the difference in behavior. As indicated in Fig. 5 we see that the visual, motor or central processing systems are possible sources of interferences. Perceptual interference (or masking), information processing limitations, a single channel central processor, a single strategy processor, increased variability, differential allocation of attention, and motor interference are possible types of interference that could lead to one-axis two-axis differences in behavior.

B. EXPERIMENTAL STRATEGY

Within the context of a classical manual control situation--quasi-random input, compensatory display, linear controlled element dynamics--it is possible to localize to some extent the source and identify the type of interference. Consider the following three control situations: (1) homogeneous dynamics and inputs--the same controlled element dynamics and the same input power spectrum in the two axes; (2) heterogeneous inputs, homogeneous dynamics--different input spectra in the two axes, but the same dynamics; and (3) heterogeneous dynamics, homogeneous inputs--different dynamics in the two axes but the same input spectra. We may consider each of these three experimental situations to have one of two possible outcomes: there either is or is not a significant difference between the single-axis performance and two-axis performance of the human controller. There are thus eight possible outcomes to the entire experimental program (some of which are highly unlikely to occur in practice). The four plausible outcomes and their implications are discussed below. The term "interaction" is used to designate a one-axis, two-axis performance difference.

1. No Interaction

It is conceivable that with sufficient training the subjects can learn to track each of two axes as well as a single axis for all three experimental conditions. If so, then we conclude that there is no motor or perceptual interference, that two control strategies can be processed as well as one, that the human controller is not information rate limited, and that increased variability (if any), has no effect on performance.

2. Interaction in all Three Control Situations

In this eventuality we shall have to examine the details of the describing function data in order to determine the source of the interaction. Information rate limitation, inability to generate simultaneously two different strategies, and other effects resulting in time sharing at one or more points should result in an increased effective time delay. Difficulty in generating two strategies may produce other modifications of the describing function when the dynamics are heterogeneous. Motor interference and random couplings in the perceptual system central processor will cause an increase in the part of the error that is uncorrelated with the input. Increased variability in the human controller's characteristics will cause an increase in the controller's remnant, and may also cause an increase in the variability of the NMSE and of the describing function parameters. If the interaction occurs primarily on the axis providing the easier task (i.e., resulting in the smaller NMSE), attentional effects are indicated. If this is the predominant effect, interaction should be greatly reduced by a redesign of the experiment that results in equal NMSE's on the two axes.

3. <u>No Interaction in the Homogeneous Situation, Interaction with Heterogeneous Dynamics or Inputs</u>

This result allows us to localize partially the source of the interference. We can conclude that the information processing rate is not significantly limited, that there is no perceptual or motor interference when the errors and movements on the two axes of the same character, and that the two information sources and processors on the two axes are similar. Sources of interference are likely to be random couplings, attentional effects, and, with heterogeneous dynamics, difficulty in generating two strategies. The relative contribution of these sources may be ascertained from the type of changes in describing functions, the extent of the increase in the uncorrelated portion of the error, etc. as described above.

4. Interaction Only with Heterogeneous Dynamics

This outcome allows us to localize the source of interference still further. Lack of interaction with heterogeneous inputs--a control situation which produces movements of different bandwidths on the two axes--minimizes the probability that random couplings are a significant type of interference. The most likely source of interference is the difficulty of generating two control strategies simultaneously. This conclusion can be tested as described in part (2) above.

We have dichotomized the outcomes of the individual experiments in order to simplify our interpretation of the experimental results. In practice we expect a continuum of interaction among the experimental conditions. From the results published in the literature and from the results of some preliminary experiments that we performed (which are discussed below), we can postulate that the experiments suggested above will show that interaction increases as the subjective difficulty of the task increases. That is, there should be little or no interaction when the control situation is homogeneous, more interference should occur when the inputs are heterogeneous, and the most interference should be seen when the dynamics are heterogeneous.

With these postulated results in mind we performed experiments with the three types of control situations just described. First, we ran an extensive set of preliminary experiments to check the validity of these postulated results and to choose the experimental parameters. Then we ran the set of three formal experiments.

C. PRELIMINARY EXPERIMENTS

The preliminary experiments are described fully in two progress reports (Ref. 11 and 12) and we give here only a brief description of them and of the results obtained. In this preliminary series we investigated the two-axis, one-axis differences with dynamics K/s^2, K/s, and K. In some of the experiments the dynamics in both axes were the same and in others they were different. Inputs of different bandwidths were employed. One highly trained subject was used. The results of these experiments indicate that with sufficient training the subject could control each axis as well in a two-axis situation as he could in a single-axis situation when the controlled element dynamics and the input spectrum on both axes were the same. However, when the dynamics in the two axes were different, we found that performance in each axis of the two axis situation was significantly worse than in the corresponding one axis situation.

D. FORMAL EXPERIMENTS

Three formal experiments were performed in which there were two main variables, the plant dynamics which were either K/s^2 or K, and the input forcing function bandwidth which was 3.5 rad/sec, 2.5 rad/sec, or 1.5 rad/sec.

1. Experiment 1: Homogeneous Control Situation

In the first experiment the dynamics in the two axes were the same and the input forcing functions were the same. The principal variable of this experiment, in addition to the one-axis two-axis primary variable was the input bandwidth. The purpose of this experiment was to test the hypothesis that the human controller could have two parallel channels in a two-axis homogeneous situation with each channel operating as well as in a single-axis control situation.

Forcing functions having rectangular spectra with cutoff frequencies of 3.5, 2.5, and 1.5 radians were investigated, in that order. The corresponding controlled-element dynamics relating stick displacement in cm. to error dot displacement in cm. were $64/s^2$, $32/s^2$, and $16/s^2$. The forcing-function spectra and the controlled-element dynamics were the same in both axes for all experiments.

2. Experiment 2: Heterogeneous Inputs, Homogeneous Dynamics

This experiment was performed to show whether or not the parallel processing capabilities of the controller depended upon the bandwidth of the movements. Forcing functions of different cutoff frequencies were employed--the low-bandwidth input (1.5 rad/sec) on the X axis and the high-bandwidth input (3.5 rad/sec) on the Y axis. The waveforms were identical to those used in the homogeneous-tracking experiment. In order to provide maximum control-display compatibility, the controlled-element dynamics were $64/s^2$ on both axes.

Two variations of this experiment were performed. The mean square of the forcing-functions for both axes were equal during the first variation. A second variation was conducted with the mean-square of the forcing-functions readjusted to produce approximately equal mean-squared tracking errors on the two axes.

3. Experiment 3: Heterogeneous Dynamics, Homogeneous Inputs

Homogeneous inputs were used in the third experiment, but the controlled-element dynamics in the two axes were different. One axis had acceleration dynamics of $64/s^2$, whereas the other had proportional dynamics of 4. The controlled-element arrangement employed for each subject is shown in Table 1. The forcing functions on both axes had cutoff frequencies of 3.5 rad/sec, their mean-squared amplitudes were equal, and the waveforms were identical to those employed in the first experiment. The purpose of this experiment was to confirm the result obtained by Chernikof et al. (Ref. 7) and by us in our preliminary experiments that heterogeneous dynamics lead to a degradation in the operator's ability to behave as a two-channel controller. In addition, we wanted to determine the extent and nature of the one-axis two-axis differences in human controller describing functions in order to develop a model for the controller that would be applicable to this kind of heterogeneous situation.

E. DESCRIPTION OF APPARATUS

1. General Description

A functional diagram of the entire two-axis tracking system is presented in Fig. 6. A linear signal-flow diagram is given in Fig. 7. The human controller was presented with a two-axis compensatory display consisting of a single error dot and a stationary reference circle. The controller attempted to keep the dot in the center of the circle by manipulating a two-axis control stick. The controlled elements for each axis were simulated on an Electronic Associates Inc. TR-48 analog computer. The input signals, i.e., disturbance functions, were provided by a multi-channel FM magnetic tape system. The X and Y components of the system output were subtracted respectively from the X and Y input signals to provide the X and Y components of the displayed error.

2. The Display

The display and control were located in a subject booth that was isolated acoustically and visually. A photograph of the subject booth is shown in Fig. 8. The display was presented on the face of an oscilloscope of 12-cm diameter. An overlayed reticle provided a rectangular array of grid lines separated by 1 cm. The subject was instructed to center the reference circle before a trial in order to correct for drifts in the system. The distance between the subject's eyes and the display was between 30 and 40 cm. The subject was allowed to choose a distance within this range that was comfortable.

3. The Hand Control

With his right hand the subject manipulated a flexible nylon stick attached to a force-sensitive hand control (Measurement Systems Hand Control, Model 435). In order to provide a high degree of control-display compatibility, the control was oriented so that the stick was horizontal and could be moved in a plane parallel to the scope face. The response of the error dot to a deflection of the stick was in the same direction as the stick motion. The nylon stick provided an omidirectional spring restraint with a restoring force of .45 kgm per cm deflection of the tip of the stick.

The stick was located 25 cm to the right and 36 cm below the center of the display scope; the tip protruded 7 cm beyond the plane of the scope face. The subject used wrist motions to manipulate the stick and he was provided with an arm rest to support his forearm.

The transducer of the hand control provided two independent electrical outputs, one proportional to the X- and the other to the Y-component of deflection. The output of the transducer in each axis was 4 volts per cm of steady-state deflection. The stick was allowed to move freely in both axes in all experiments. In the single-axis experiments the error dot in the inactive axis was clamped electronically at zero displacement.

4. Controlled-Element Dynamics

The dynamics of the controlled element were either proportional (K), or acceleration (K/s^2). The gains of the controlled elements were adjusted so that the control effectiveness would be roughly the same for all bandwidth conditions of Experiment 1 (homogeneous control situation). In Experiment 3 (heterogeneous dynamics), the gains were adjusted so that the control effectiveness in each axis would be the same as in Experiment 1. The control effectiveness was defined for an axis having acceleration dynamics as the maximum error dot acceleration obtainable divided by the rms acceleration of the forcing function. For proportional controlled-element dynamics, control effectiveness was the ratio of the maximum error dot displacement obtainable to the rms displacement of the input. Since the control stick had no mechanical stops to limit its deflection, the maximum stick deflection was arbitrarily chosen as 3 cm for purposes of calculation. This deflection was never exceeded during the experimental trials.

5. Forcing Functions

The input signals were pseudo-Gaussian with augmented rectangular power spectra. Each input signal contained a primary and a secondary component as shown in Fig. 9. Both components were constructed by summing 40 or more sinusoids of equal amplitudes spaced linearly in frequency. Therefore they had essentially flat power spectral densities extending from slightly above zero rad/sec to their respective cutoff frequencies. The cutoff frequency of the primary signal, ω_1, was either 1.5, 2.5, or 3.5, rad/sec. The cutoff frequency of the secondary component was 9 rad/sec in all cases. The power level of the secondary component was 26 db below that of the primary component. Preliminary experiments indicated a secondary signal would permit valid measurements beyond the frequency range of the primary signal without affecting very much the low-frequency behavior of the human controller. The X and Y input signals were linearly uncorrelated in all cases.

In Experiments 1 and 3 and in the first variation of Experiment 2 the mean-square deviation of the forcing function was 4.0 cm^2 on each axis. Thus, the total (i.e., X plus Y) mean squared amplitude was 4 cm^2 for either single-axis task and 8 cm^2 for the two-axis task. In the second variation of Experiment 2, in which we adjusted the input magnitude to produce approximately equal errors in the two axes, mean-squared input levels were 6.9 and 1.1 cm^2, on the X and Y axes, respectively. The two-axis total input level was thereby maintained at 8 cm^2.

6. Knowledge of Performance

The subjects were instructed to minimize the mean-squared tracking error. In order to encourage the subjects to adopt this criterion, they were given knowledge of their performance in two ways. A delayed indication was provided in terms of the performance scores, which were available for inspection after each block of trials. Performance was measured in terms of normalized mean-squared error for each axis tracked. Complete histories of the performance of all subjects were posted and shown to each subject in an attempt to foster a spirit of competition.

Continuous feedback of the subject's performance was also provided by variations in the diameter of the reference circle. The instantaneous circle diameter D(t) was approximately

$$D(t) = \frac{1}{T} \int_{t-T}^{t} [K_x e_x^2(t) + K_y e_y^2(t)]\, dt + D_o \qquad (3)$$

where K_x and K_y were weighting factors applied to the X and Y errors, and D_o, the minimum diameter, was set to 0.3 cm. The short-term averaging was approximated by a first-order low-pass filter having a time constant of 10 seconds.

In order to provide uniform inventive to the subjects, the weighting constants K_x and K_y were adjusted inversely to the difficulty of the task so that the circle diameter was roughly 0.6 cm on the average for all experimental conditions. The proper settings of the weighting constants for the single-axis experimental runs were determined on the basis of the single-axis performance. When the subject tracked both axes simultaneously, K_x and K_y were readjusted so that (1) the average circle diameter in the two-axis task would be the same as in each of the single-axis tasks (provided that the subject performed as well on each axis in the two-axis task as in the single-axis task), and (2) the errors on the two axes would contribute equally to the circle diameter. Thus, if K_x and K_y were the weighting constants needed to maintain an average circle diameter of 0.6 cm in single-axis tracking, the weighting constants in the two-axis task would be $K_x/2$ and $K_y/2$. This procedure was adopted to encourage the subject to attend equally to the two axes even though the X and Y tasks might be unequal in difficulty. There was one exception to this rule: K_x and K_y were made equal for the homogeneous control situation in order to simplify the experimental procedure even though there were X- and Y-axes differences.

F. SUBJECTS

Three subjects participated in the first experiment, two in the second, and three in the third, as shown in Table 2. All subjects were college students without flight experience who received extensive training in the tracking situations tested in these experiments. Most of the training was in a two-axis situation with K/s^2 dynamics. As the subjects became more proficient the forcing function bandwidth was increased to increase the task difficulty to the level that they would encounter in the actual experiments. A typical record of the normalized mean-square error during training of one subject is shown in Fig. 10. The training period extended over 1-1/2 months before Experiment 1 was run, during which time the subjects received approximately fifteen hours of tracking practice. During this period their performance improved markedly and appeared to approach a fairly stable level. When the control situation was changed to include new dynamics or to be heterogeneous, the subjects received more training in the new situation until their performance appeared to become stable.

G. PROCEDURE

Data taking required a total of nine 4-minute trials per subject per experimental condition. The trials were presented in a balanced order as shown by the experimental plan outlined in Table 3. The trials were grouped into three sessions, each of which contained an X-axis, an Y-axis, and a two-axis trial. The sessions were separated by 15-minute rest periods, and the trials within a session were separated by 1-minute rest periods. Each axis condition was included in each session to allow the pairing of performance scores and thereby reduce the influence of learning and fatigue on the experimental results.

Three performance scores were obtained during each 4-minute trial. The middle 3-minutes of each trial was divided into scoring periods of 48 sec, separated by intervals of 18 secs for readout and for resetting the scoring integrators.* The subject, however, was unaware of the scoring times and tracked continuously for the entire 4 minutes.

Since the input signals were provided via a tape system, the same 4-minute segment could be presented repeatedly to a subject. This was done in order to minimize experimental variation and to determine the dependence of the operator's behavior on the particular waveshape of the forcing function. Thus, only one pair of input waveforms (one for X and the other for Y) was used for each experimental condition. Each waveform was tracked six times during the course of data taking--three times alone and three times simultaneously with the other input. Learning of the waveform by the subject was not expected to influence the experimental results significantly. That is, although the scores were expected to be somewhat lower than if the subject had been tracking the output of an on-line random noise generator, we assumed that learning would not affect the differences, if any, between single- and two-axis performance.

H. PERFORMANCE AND DESCRIPTIVE MEASURES

Mean-squared error scores were computed to provide an indication of overall system performance. Human controller

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* A slightly different experimental plan was followed for subject EK during the preliminary phase of the experiment. Twenty-seven trials were recorded, as outlined in Table 4, and a single measure was obtained per trial.

describing functions were also obtained to provide a description of the controllers' characteristics. The techniques used to compute these measures, the calibration of the measurement techniques, comparisons between our results and those obtained by McRuer et al (Ref. 3), and the variability of our results are discussed in detail in Section IV.

IV. ANALYSIS TECHNIQUES

A. DESCRIPTIVE MEASURES

1. Normalized Mean-Squared Error

Normalized mean-squared errors (NMSE) were computed for each axis individually and for the two-axis task as a whole. The NMSE for an individual axis was obtained by dividing the mean-squared tracking error on that axis by the corresponding mean-squared input deviation. When two axes were tracked simultaneously, two such independent measurements were obtained, one for each axis. The error scores obtained during two-axis tracking were also combined to yield single, total-task NMSE. This measure was computed by dividing the total squared error by the total squared input. In order to have a combined-axis measurement relating to single-axis performance, the sum of the mean-squared errors obtained from the X and Y axes tracked singly was divided by the sum of the X and Y mean-squared inputs. This measurement can be interpreted as the "predicted" two-axis performance; that is, it would be equivalent to the two-axis measurement if the subject performed equally in the two-axis and one-axis situations.

2. Describing Functions

Human controller describing functions relating centimeters of stick displacement to centimeters of error displacement were obtained using a multiple regression analysis technique described in earlier reports (Ref. 13 and 14). This technique employs a model composed of a linear combination of a set of orthogonalized exponential filters to represent the human controller's characteristics. The parameters of the model are given in Appendix A. The input signals used in our experiments extended out to 9 rad/sec and thus provided data on the describing function out to this frequency. In most of our measurements the tracking error was used as the input to the model and the human controller's stick movement was matched by the output of the model. Such error-to-stick measurements provided all the describing function data when the controlled element was K/s^2. When the controlled element was K, error-to-stick measurements provided describing function data at frequencies only up to 4 rad/sec. In order to avoid inaccuracies at higher frequencies resulting from circulating remnant, additional describing functions were determined which related system input to system output. A closed-to-open-loop conversion of these measurements provided describing function data for the human controller at frequencies above 4 rad/sec.

B. CALIBRATION

1. Prewhitening

With K/s^2 dynamics it was necessary to pre-whiten the error and stick signals used for computing the human controller's describing function by passing these signals through a single stage, low-pass filter implemented on the digital computer. Even though the input forcing function was essentially flat up to the cut-off frequency, the controlled-element dynamics of K/s^2 resulted in a stick spectrum that increased at approximately 12 dB per octave. Since the analysis technique finds the describing function whose output matches the stick signal with least mean-squared error, the describing function obtained when the stick signal has most of its power concentrated at high frequencies could differ greatly from the controller's actual describing function at low frequencies without significantly affecting the match between model output and human controller output. Pre-whitening with low-pass filters reduced the high frequency of stick signal. Consequently, the measured describing function matched the controller's actual describing function over a wider frequency range.

2. Validation of Describing Function Measurements

To verify that the regression analysis techniques developed by Elkind et al (Ref. 13 and 14) were capable of measuring the kind of human controller describing functions likely to be encountered in this experiment, a number of validation measurements were taken. In one set of these measurements a manual control system with $C(s) = K/s^2$ was simulated on an analog computer and an analog filter was substituted for the human controller. The transfer function of the analog test filter was of the form $K(s+s_1)/(s+s_2)$, where $s_2 > s_1$. McRuer et al (Ref. 3) have shown that except for the absence of a time delay, this is an appropriate representation of human controller characteristics for controlled-element dynamics of K/s^2.

An initial test was conducted to determine the ability of our analysis procedures to reproduce the transfer function of the analog filter under optimum measurement conditions, that is, with a wide-band pseudo-Gaussian noise source driving test the filter directly. The test filter was not embedded in a feedback loop. The transfer function of the test filter was

$$F(s) = \frac{(s + 1/4)}{(s + 4)} \tag{3}$$

The forcing function was one of those used in our experiments and had a bandwidth of 3.5 rad/sec. The signals were not pre-whitened.

The experimental and theoretical transfer functions are compared in Fig. 11. The mean-squared difference between the outputs of the model and the test filter was less than 1%. The difference between theoretical and computed amplitude ratios was less than 2 dB over the entire measurements range of 1/16 to 16 rad/sec. The difference between theoretical and computer phase shift was less than 10 degrees between 1/16 and 4 rad/sec and incremented at higher frequencies. The large (20°-40°) high frequency phase difference result from the fact that the orthonormal filters used in the analysis procedure had poles at negative real frequencies of .33, 1, 3, 9, and 27 rad/sec. Such a filter set will have a phase lag of at least 30° at 16 rad/sec because of the presence of the pole at $s = -27$. In the case of the human controller, this additional lag should not be the source of appreciable error because the human's neuro-muscular lag and time delay will introduce phase lag at high frequencies. These sources of lag were not included in our test filter.

A subsequent test was performed to evaluate the analysis techniques under measurement conditions more appropriate to the experimental situation. The test filter representing the human controller was placed inside a simulated control loop so that its input and output waveforms would be similar in spectral content to the error and stick signals obtained during tracking with K/s^2 dynamics. The transfer function of the test filter was

$$F(s) = \frac{1}{2} \frac{(s + 1/2)}{(s + 4)} \tag{4}$$

The system was excited by the same forcing function used in the preceding test. The model had poles at negative real frequencies of 2, 2.8, 4, 5.7, and 8 rad/sec. This set of filter poles was used to model some of the human controller describing functions. The filter was analyzed under two conditions: (1) with "remnant" (uncorrelated noise) added to the simulated stick output, and (2) without "remnant". The signals were pre-whitened for analysis in both cases. The additive noise term was provided by a recording of the control motions of a human tracker obtained under similar experimental conditions. Since the forcing function corresponding to the human data was uncorrelated with the one driving the analog network, the noise component was linearly uncorrelated with that portion of the filter response elicited by the forcing function. The mean squared amplitude of the analog remnant was adjusted to be about 30% of the mean squared amplitude of the total response of the filter simulating the human controller.

The measured describing functions obtained in this validation experiment are compared with the calculated response of the test filter in Fig. 12. Without remnant the measured describing function was able to account for 99% of the output power of the analog filter. Figure 12 shows that the range of frequencies over which the measured and calculated amplitude ratios differed by 6 dB or less was 1/2 to 16 rad/sec for the simulation without remnant and 1/4 to 16 rad/sec when remnant was included. Amplitude ratios measured under both conditions were within 3 dB of the computed values between 1/2 and 4 rad/sec. Differences of less than 25 degrees between measured and computed phase shift were found between 1 and 8 rad/sec for the simulation without remnant and between 1/2 and 4 rad/sec for the simulation with remnant. If we consider that most of the phase errors at 8 and 16 rad/sec would have been eliminated if we had included an additional lag and time delay in the test filter, it appears that reasonable measurements of the human controller's describing function can be obtained between 1/2 and 16 rad/sec. Very accurate measurements can be obtained in the range of 1 to 4 rad/sec, wherein lies most of the tracking error power.

We can also calibrate our analysis techniques by comparing the measurements obtained with our technique with the results obtained by other investigators using different methods. In Fig. 13 is a plot of the open loop describing function (human controller plus plant dynamics) obtained with dynamics of $K's^2$ and an input forcing function of 2.5 rad/sec. Also plotted on the figure is an open loop describing function obtained by McRuer et al (Ref. 3) with the same dynamics and a similar input forcing function. Standard deviations of amplitude ratio and phase shift at selected frequencies are indicated by brackets. Note that our results and McRuer's are of the same form. In particular, we both obtain approximately the same crossover frequency, ω_c, of about 4 rad/sec. The principal difference in the two sets of results is that our low frequency amplitude ratio is about 10 db higher than McRuer's.

Another comparison between our results and McRuer's is provided by the relationship between mean-squared error and input forcing function bandwidth. McRuer shows that the normalized mean-squared error is related to the cutoff frequency of the input forcing function by the so-called one-third law

$$\frac{\sigma^2_e}{\sigma^2_i} = \frac{1}{3} \left(\frac{\omega_i}{\omega_e}\right)^2 \qquad \omega_c > > \omega_i \tag{5}$$

where ω_i is the bandwidth of the input signal and w_c the open loop gain crossover frequency. The values of NMSE predicted by the one-third law are compared with the values obtained experimentally in Fig. 14. ω_c was taken to be 4 rad/sec for the theoretical calculations. The experimental values used in the figure are the averages over three subjects of the NMSE in a single axis control system with K/s^2 dynamics. On the whole the agreement between the theoretical and measured values is good.

3. Relation Between Describing Function and NMSE

The relation between the NMSE and $H(j\omega)$ is generally complex and depends on the precise shape of $H(j\omega)$, the controlled dynamics $C(j\omega)$, and the input spectrum $I(\omega)$. Of greater importance to the context of this experiment is the relation between changes in $H(j\omega)$ and changes in NMSE under conditions in which $I(\omega)$ and $C(j\omega)$ are invariant. A simple relationship can be derived if we assume that:

1. The magnitude of the open-loop transfer function $HC(j\omega)$, in linear units, is much greater than unity.

2. The operator adopts the same strategy in one- and two-axis situations; that is, the Bode plots differ only by a constant shift in gain.

3. The amount of error not correlated with the input is roughly proportional to the correlated error.

The power density spectrum of the mean-squared tracking error, $S_e(j\omega)$ is

$$S_e(j\omega) = \frac{S_i(\omega)}{|1 + HC|^2} \doteq \frac{S_i(\omega)}{|HC|^2} \qquad (6)$$

where $S_i(\omega)$ is power density spectrum of the input. Only that portion of the error correlated with the input signal has been considered. Let $H_1(j\omega)$ be the human-operator describing function appropriate to 1-axis tracking, and let $KH_1(j\omega)$ be that corresponding to 2-axis tracking. The ratio of two-axis to one-axis mean-squared errors is then

$$\frac{\sigma^2_e|_2}{\sigma^2_e|_1} \doteq \frac{1}{K^2} \tag{7}$$

Expressed in dB the difference between human-operator describing functions is

$$20 \log K = -10 \log \frac{\sigma^2_e|_2}{\sigma^2_e|_1} \tag{8}$$

Thus, decreases in gain of 1, 2, and 3 db should correspond respectively to increases in NMSE of 26%, 58%, and 100%.

C. VARIABILITY OF RESULTS

1. Expected Trend in Variability

McRuer et al (Ref. 3) have shown that the human controller generates a describing function that is highly repeatable over a confined portion of the spectrum when tracking with a controlled element of K/s^2. The range of high repeatability generally spans a two to three octave band lying immediately below the gain-crossover frequency. The variability of repeated measurements of the describing function outside this frequency range is relatively large. A reasonable explanation for the observed behavior is that the human controller tries to maintain a tight control over his describing function only in the frequency region that is critical to good system performance. It is easy to show that when

$\omega_i < \omega_c$ over 98% of the error power is contained within the frequency range of $\omega_i/4$ to ω_i. Therefore, poor control over the transfer function outside this region will have little effect on the mean squared tracking error (provided that a reasonable phase margin is maintained).

In summary, we should expect to obtain highly repeatable measurements of the describing function over the highest two octaves of frequency spanned by the forcing function. On the other hand, we expect variable results outside this range because (1) the reliability of the measurements is degraded for K/s^2 tracking because of the relative lack of control power and (2) good system performance does not require tight control of the describing function outside the critical two-octave frequency band.

2. Intrasubject Variability

Single-axis describing functions for a typical subject are presented in Fig. 15. Each value of amplitude ratio and phase shift is the mean of three measurements. The standard deviations of the measurements are indicated by brackets. The controlled-element dynamics were either K (Fig. 15a) or K/s^2 (Fig. 15b). The bandwidth of the forcing function was 3.5 rad/sec.

The repeatability of the measurements was relatively high for both tracking situations. The standard deviation of the amplitude ratio measurements was 3 db or less at all measurement frequencies for both proportional and acceleration tracking, and the standard deviation of the phase shift was less than 15 degrees. In the critical frequency range of 1-4 rad/sec (and, in fact, up to 16 rad/sec) the standard deviation of the amplitude ratio was less than 1 db.

3. Intersubject Variability

Intersubject variability is illustrated in Fig. 16. Each describing function shown is the mean of three single-axis describing functions, each from a different subject. The describing functions correspond to proportional tracking (Fig. 16a) and acceleration tracking (Fig. 16b).

The variability among subjects was greater than the variability within subjects, although the repeatability over the critical frequency region was good. For acceleration tracking, the standard deviations of the amplitude ratio and phase shift were less than 3 db and 10 degrees, respectively, at frequencies above 1/2 rad/sec. The standard deviation of the phase shift was considerably larger at lower frequencies, owing to the anamolous response of one subject. The range of good repeatability was greater for proportional tracking. The standard deviation of the gain was less than 3 db at all frequencies above 1/8 rad/sec; the standard deviation of the phase shift was less than 35 degrees over the entire measurement range.

V. EXPERIMENTAL RESULTS

A. EXPERIMENT 1: HOMOGENEOUS CONTROL SITUATION

1. NMSE Scores

Tables 5 through 7 indicate quantitatively the difference between one-axis and two-axis performance. The levels of performance for each subject under each axis-bandwidth condition, for one- and two-axis tracking separately, are presented in Table 5. Each of these entries represents an average NMSE based on nine data points, three scores for each of three runs. Also shown for each condition are the average performance levels of the three subjects.

The percent difference between the one- and two-axis scores for each subject under each bandwidth-axis condition are given in Table 6. Each entry represents the percent change in NMSE on a given axis caused by the addition of a simultaneous tracking task on the other axis. The percent change is defined as:

$$\Delta \text{NMSE} = \frac{(\text{2-axis NMSE}) - (\text{1-axis NMSE})}{(\text{1-axis NMSE})} \times 100 \qquad (9)$$

A positive change indicates that one-axis performance was superior. The average ΔNMSE of the three subjects for each bandwidth-axis condition are also indicated in the table. Each entry is the average of twenty-seven scores.

Table 7 shows the average percent change in NMSE for each subject averaged across the three bandwidth conditions. Each entry is the mean of twenty-seven data points. Also shown are the average changes under each axis condition averaged over the three subjects.

A three-factor analysis of variance (Ref. 15) was performed on the error scores for each axis-bandwidth condition. There were three bandwidth conditions (1.5, 2.5, and 3.5 rad/sec cutoff frequencies) and three axis conditions (X, Y, and total-task), requiring nine separate analyses. The three analysis factors were (1) number of axes tracked simultaneously (i.e., one- or two-axis tracking), (2) subject, and (3) input segment. There were three replications of each number-subject-segment condition. A factorial representation of the data is presented in Fig. 17. The analyses of variance are presented in Tables B1 to B9 of Appendix B. The NMSE scores were scaled for computational purposes, as indicated in each table.

The primary purpose of the analysis of variance was to test the null hypothesis that the average two-axis NMSE was not significantly different from the average one-axis NMSE. An equivalent statement is that the primary quantity of interest is the significance of the F-ratio pertaining to differences along the number-of-axes dimension. Differences along this dimension will be referred to as the "number effect". Differences in tracking proficiency among subjects and differences in the difficulty of each segment of the forcing function are of secondary interest.

In order to test the significance of the number effect, it was necessary to test first the significance of the interactions between number of axes tracked and subject and between number and segment. A significant interaction of either of these types implies that the change in performance caused by the addition of a second axis of tracking varied significantly among subjects or among segments. The variance of the number effect was tested against the variance of whichever interaction was significant. If there were no significant interactions, the variance of the number effect was tested against the experimental (i.e., "within cells") variance. The number effect sometimes failed to be significant because of a significant subject-number interaction. That is, although one or more of the subjects individually showed significant number effects, the one-axis two-axis differences varied so greatly among the subjects that no statistical significance would be attached to the behavior of the population as a whole.

The data corresponding to the 3.5 rad/sec input bandwidth are summarized in Tables B1 to B3. Number effects were not significant on the X axis, significant on the Y axis at the 0.001 level, and significant for the total task at the 0.01 level. There were no significant interactions.

Tables B4 to B6 summarize the experiment with the 2.5 rad/sec forcing function. No significant number effects and no significant interactions appeared on the X axis or in the total task. There was a significant (0.05 level) subject-number interaction on the Y axis. When tested against the interaction variance, the main number effect failed to be statistically significant.

The data corresponding to the 1.5 rad/sec forcing function are summarized in Tables B7 to B9. Significant number effects occurred on the Y-axis and in the total task, but not on the X axis. There were no significant interactions.

The experimental conditions which yielded significant one-axis, two-axis differences in NMSE are indicated in Table 6. The significance of the entries representing the average behavior of the three subjects was determined by the analyses of variance previously described. The significance of the results for each subject individually was computed as follows: (1) each NMSE score obtained from a single-axis trial was subtracted from the corresponding score obtained during two-axis tracking; nine such paired-difference scores were computed for each subject-axis-bandwidth condition; (2) the hypothesis that the mean of the paired differences was zero was tested by a two-tailed Student's t-test. Pairing of the scores in this manner eliminated segments and replications as first-order sources of variance. The sensitivity of the experimental procedure was such that an average fractional change of about 15% would be significant at the 0.05 level.

Table 6 shows that the percent change in total task NMSE did not increase with input bandwidth. For bandwidths of 1.5, 2.5, and 3.5 rad/sec, respectively, the average changes were 10% (significant at the .05 level), 4% (not significant), and 13% (significant at the .01 level).

As indicated by the analysis of variance, the changes were greater on the Y axis than on the X axis. One-axis two-axis differences ranged from 3% to over 40% on the Y axis and were consistently significant for two of the subjects. On the other hand, changes on the X axis were less than 15% for all subject-bandwidth conditions and were not statistically significant.

Differences between the X-axis and Y-axis performance degradations are summarized in the botton row of Table 7, which shows the percent changes averaged over all subject-bandwidth conditions. Whereas two-axis tracking caused a 24% increase in NMSE on the Y axis, a change of only 1% was observed on the X axis. The average total-task increase was 9%.

Differences between subjects are also shown in Table 7. Subject RL showed changes of 10% or less for the X axis, Y axis, and total-task performance. While subject CP also showed a change of less than 10% in total task performance, he showed a 10% decrease in NMSE on the X axis and a 27% increase on the Y axis in the two-axis situation. The remaining subject, BL, showed the largest change in total-task performance (16%) and also showed a considerably greater degradation on the Y axis than on the X axis.

2. Describing Functions

Average human-controller describing functions are shown for each bandwidth condition in Figs. 18 to 20. Each describing function is the average of three describing functions, one for each subject. All of these measurements were made on the Y axis, on which the differences between one- and two-axis NMSE's were the greatest.

Figure 18a shows the average one-axis and two-axis* describing functions for the 3.5 rad/sec input bandwidth condition. The difference between these two describing functions is shown in Fig. 18b. Figures 19 and 20 show the describing functions for the 2.5 and 1.5 rad/sec input bandwidth conditions. A t-test revealed that there were no significant differences between corresponding one-axis and two-axis describing functions. In general, the differences between one- and two-axis amplitude ratios were about 1 dB, and the phase differences were about 5°. These small differences are in accord with the small one axis-two axis percent differences in NMSE.

3. Summary

There were statistically significant but generally small changes in normalized mean squared error, with most of the change occurring on the Y axis. The deleterious effect of two-axis tracking did not increase monotonically with the task difficulty. The average fractional increase in NMSE, averaged over all subjects and bandwidth conditions, was 9% for the total task measurement. Changes in the describing functions were correspondingly small and of no statistical or practical significance.

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* The term "two-axis describing function" refers to a describing function measured under two-axis tracking conditions. The describing function is a one-dimensional descriptor which relates a single input variable to a single output variable.

B. EXPERIMENT 2: HETEROGENEOUS INPUTS, HOMOGENEOUS DYNAMICS

1. NMSE Scores

The average NMSE for each subject-axis combination are shown in Table 8. Also shown are the percent changes in NMSE caused by the addition of a second axis of tracking. Each entry is the mean of nine measurements. Asterisks signify entries that are significantly different from zero as determined by a t-test.

The top two rows of Tables 8a and 8b were obtained with equal forcing-function levels on the X and Y axes. The third row was obtained from a redesign of the experiment with subject RL. The data averaged in the third row were obtained from subject RL after the input levels were readjusted to provide nearly equal mean squared errors in the X and Y axes. The resulting mean squared input levels were 6.9 and 1.1 cm^2, respectively, on X and Y. The mean squared input level for the total two-axis task was fixed at 8.0 cm^2 for all experiments.

Tables B10 and B12 contain analyses of variance for X, Y, and total-task measurements using data obtained from subject CP and from the first experiment with subject RL. There were no significant number effects. Table B10 shows that there was a highly significant subject-number interaction on the X axis, on which the input bandwidth was 1.5 rad/sec.

Let us consider first the results of the experiments in which the X and Y forcing functions were equal. Both subjects showed changes of less than 5% for the total task; neither of these changes was statistically significant. There was a noticeable difference, however, in the way each subject modified his performance when tracking two axes simultaneously. Subject CP showed negligible changes in NMSE on both axes. Subject RL showed statistically significant changes on both axes. His NMSE decreased by 16% on the high-bandwidth axis (the Y axis), whereas his X-axis NMSE nearly doubled in the two-axis situation. The small effect that this relatively large increase in X-axis error had on total task performance was due to the existence of much larger errors on the Y axis. When subject RL tracked the two axes simultaneously, only 25% of the total mean squared error appeared on the X-axis.

The amplitudes of the forcing functions were readjusted as explained above in order to show that subject RL was influenced by the relative magnitudes of the X and Y error signals. The results of this modification are shown in the bottom row of Table 8b. Under these conditions the changes in performance induced by two-axis tracking were less than 10% on each axis and were not statistically significant. As before, there was no significant change in the total-task performance.

2. Describing Functions

Human-controller describing functions for each subject for X and Y separately are shown in Figs. 21 to 24. Each Bode plot is the result of a single 100-sec measurement. Each figure contains corresponding one- and two-axis describing functions as well as a plot of the difference between them.

The form of the describing function was approximately the same for one- and two-axis tracking on a given axis. Figures 22 and 24 show that the changes in amplitude ratio on the high-bandwidth axis were generally less than 2 dB and were not consistent in direction. Phase changes were less than 15°. Figures 21 and 23 show that two-axis tracking caused a decrease in amplitude ratio of 2 to 3 dB on the low-bandwidth axis over the range of significant error power (0.5 to 1.5 rad/sec). The corresponding phase changes were less than 15°. The decrease in amplitude ratio was expected on the low bandwidth axis for subject RL because of the correspondingly large performance degradation on that axis. A change of the magnitude observed for subject CP on the low-bandwidth axis was unexpected, since the change in NMSE was very small. One of the two describing functions used to compute this difference may have been from an unrepresentative sample. The significance of these differences was not tested since only one describing function per subject was computed.

3. Summary

Neither subject showed a significant change in normalized mean squared error for the total-task measurement. One subject showed no significant changes on either the X or Y axis. The other subject showed a performance degradation of almost 100% on the low-bandwidth axis that was effectively offset by a 16% improvement in performance on the other axis. Both changes were statistically significant. This same subject showed no significant changes on either axis when the forcing-function amplitudes were readjusted to yield the same mean squared tracking errors on both axes. The primary change in the describing function was a lowering of the amplitude ratio on the low-bandwidth axis.

C. EXPERIMENT 3: HETEROGENEOUS DYNAMICS, HOMOGENEOUS INPUTS

1. NMSE Scores

Table 9 shows the performance levels and percent changes for the three subjects individually and collectively. Entries for the individual subjects are the average of nine measurements; twenty-seven measurements were used to compute the collective means. Significant differences are designated by asterisks.

Tables B13 to B15 contain analyses of variance of the normalized mean squared error for the proportionally-controlled axis (the "K axis"), for the axis with acceleration control (the "K/s^2 axis"), and for the total task.* The subject-number interactions were significant at the .001 level for all three axis conditions. The one-axis, two-axis differences, therefore, varied greatly among the three subjects. When tested against the interaction variance, only the K-axis number effect was statistically significant.

Table 9 shows that the addition of a second axis of tracking caused a consistent degradation in performance. That

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* Since only one score was obtained per trial for subject EK, forcing-function segment has been eliminated as a dimension of the analysis.

is, the average two-axis NMSE was significantly greater than the corresponding average one-axis NMSE for each subject for the K, K/s^2, and total-task measurements. Although the direction of the effect was the same for all subjects, the magnitude was not--hence, the significant interactions and non-significant main effects.

The average increases in NMSE were 98%, 62%, and 69% for the K-axis, K/s^2-axis, and total-task measurements, respectively, for the subjects collectively. The change in total-task performance was weighted more heavily by changes in the K/s^2 axis than by changes in the K axis because of the difference in mean squared errors on the two axes. On the average, the K-axis errors accounted for only 25% of the total mean squared error during two-axis tracking.

There were large differences in strategies among the individual subjects. The subject who showed the smallest total-task degradation (38%), showed the least change on the K/s^2 axis (13%) and the greatest change on the K axis (124%). The subject showing the greatest total-task change of 87% performed in a similar way on the two axes. His NMSE increased 110% on the K axis and about 80% on the K/s^2 axis. The remaining subject showed a smaller percent change on the K axis (59%) than on the K/s^2 axis (91%), while suffering a total-task degradation of 81%.

Tracking ability improved throughout the course of the experimental program. Subjects CP and RL achieved an average NMSE of 0.227 for one-axis acceleration tracking on the Y axis during the first experiment. When tracking the same forcing function with the same controlled-element dynamics during the final experiment, the same two subjects achieved an average NMSE of o.128.

2. Describing Functions

Describing functions for proportional and acceleration tracking are shown in Figs. 25 to 32. Figures 25 and 29 show the behavior of the subjects collectively, whereas the remainder of the Bode plots show the behavior of the subjects individually. The individual-subject describing functions are the average results of three replications. The resulting average describing functions, one per axis per subject, were used to compute the average collective behavior. Amplitude ratio and phase differences significant at the .05 level, as determined by a t-test, are indicated by asterisks beside the points plotted in part b of each figure. Standard deviations of the amplitude ratio and phase shift at selected frequencies are indicated in the figures by brackets.

Open-loop describing functions (human controller cascaded with the controlled element) are presented for data obtained from proportional tracking so that the gain crossover frequencies and phase margins may be determined by inspection. The human controller's describing function has the same critical frequencies and time delay as the open-loop describing function and may be derived from the latter by subtracting 12 dB from the amplitude ratio at all frequencies. Human controller describing functions are presented for acceleration tracking.

There are at least two kinds of changes that we might expect the two-axis situation to induce in the human controller's describing function. One kind of change--one that we have seen in the previous two experiments--is a relatively uniform lowering of the amplitude ratio which will result in larger tracking errors. Another type of change that we might expect from tracking with heterogeneous controlled element dynamics is a change in the human controller's equalization (i.e., critical frequencies and time delay). A change in equalization will probably (but not necessarily) degrade system performance.* In general, one might expect to see both a change in equalization and an overall shift in the amplitude ratio.

The describing functions have been approximated by transfer functions of the following form:

$$H(s) = \frac{(T_L s+1)\, e^{-\tau s}}{(T_{I'} s+1)(T_I s+1)(T_N s+1)} \qquad (10)$$

τ and T_N represent the time delay and neuromuscular lag discussed by McRuer et al (Ref. 3). The remainder of the model is a departure from that of Eq. (2), which was derived from Ref. 3, in that we have added a second lag term $(T_{I'} s+1)$.

* If the controller is able to adopt the optimum equalization when tracking a single axis, any change in equalization caused by two-axis tracking must degrade performance. However, if his equalization is not optimal in the one-axis situation, he may be able to track just as well with some other non-optimal equalization in the two-axis situation.

This lag affects only the low frequency response, and $1/T_{I'}$ was always 1.0 or less rad/sec. This modification is necessary to provide an adequate approximation to our describing functions, some of which exhibit a low-frequency lead-lag behavior.

One of the models postulated by McRuer et al (Ref. 3) and Magdaleno (Ref. 16) contains a low-frequency lag-lead network to account for the low-frequency phase lags observed by the authors. They attribute this phenomenon to properties of the neuromuscular system. However, our data indicate a lead-lag (rather than a lag-lead), which is not likely to result from the same mechanism. The lead-lag behavior observed by us may have been introduced because the subject was required to track with both K and K/s^2 dynamics.

The parameters of the approximations are tabulated in Table 10. We found that the high-frequency behavior of all the K-axis describing functions were adequately simulated with $\tau = .09$ sec and $1/T_N = 16$ rad/sec. The value for τ is the same as that used by McRuer et al (Ref. 3). The value for T_N is somewhat less than his value, which was approximately .11 for a tracking situation in which $C = K/(s-a)$. It is, however, within the range of values obtained by Elkind (Ref. 1) in his studies of systems in which the controlled element was a simple gain K. In addition, a lag T_I of 0.4 sec, corresponding to a critical frequency of 2.5 rad/sec, appeared in all K-axis describing functions. All variations occurred in the lead-lag network represented by time constants T_L and $T_{I'}$.

Figure 25 shows the describing functions averaged for the three subjects for the K axis. Low-frequency lead appeared in both the one-axis and two-axis describing functions. Two-axis tracking caused a decrease in amplitude ratio at all frequencies below 8 rad/sec. These differences ranged from 8 dB at 1/16 rad/sec to 2 dB at 4 rad/sec and were statistically significant between 1 and 4 rad/sec. Over this frequency range, the differences were approximately 3 dB, which is what we would expect from the doubling of the NMSE. The two-axis situation also caused an increase in phase shift of up to 20 degrees at frequencies below 2 rad/sec, but these differences were not statistically significant.

There is a corresponding difference in the analytic approximations. Whereas $1/T_L$ remains constant at .25 rad/sec, $1/T_{I'}$ increases from 0.5 rad/sec for one-axis tracking to 1 rad/sec for two-axis tracking. This increase in $1/T_{I'}$, coupled with a lowering of the gain K_h, accounts for much of the difference between the one- and two-axis describing functions.

There is, however, a discrepancy at high frequencies between the measured describing functions and the analytical approximations that will affect our interpretation of the one-axis two-axis differences. The negligible changes in measured phase shift at the gain crossover frequency (8 to 10 rad/sec) indicate that the controller did not increase his phase margin for the two-axis task. The analytic approximations, on the other hand, indicate an increase in phase margin of about 10 degrees resulting from the lowering of the amplitude ratio at high frequencies.

Figures 26 to 28 contain the K-axis describing functions for the individual subjects. Each subject showed a statistically significant decrease in amplitude ratio over part of the spectrum under two-axis conditions. In addition, two of the subjects changed their low-frequency equalization.

Subject RL (Fig. 26) showed a statistically significant decrease in amplitude ratio at all but one measurement frequency below 8 rad/sec. This difference was a maximum of 13 dB at 1/16 rad/sec, decreased monotonically to 2 dB at 1 rad/sec, and increased to around 4 dB at 2 and 4 rad/sec. There were increases in the phase shift to 20 to 40 degrees between frequencies of 1/8 to 1/2 rad/sec.

The analytic approximations differ in the values chosen for T_L and $T_{I'}$. An approximation to the single-axis describing function has been derived with T_L and $T_{I'} = 0$. Critical frequencies of $1/T_L = .33$ and $1/T_{I'} = 1$ rad/sec are necessary in order to approximate the low-frequency lead-lag behavior of the two-axis describing function.

Figure 27 shows that subject EK also lowered his amplitude ratio in the two-axis situation and, to some extent, modified his equalization. The amplitude ratio decrease was significant at all frequencies below 4 rad/sec and ranged from 4 to 8 dB. There was an increase in the phase shift at all frequencies below 16 rad/sec. Significant phase differences of 25 to 30 degrees occurred at 2 and 4 rad/sec.

The analytical approximations to the one- and two-axis describing functions both contain a low-frequency lead-lag behavior and differ in the value chosen for the zero frequency ($1/T_L$). Whereas $1/T_{I'}$ is fixed at 1 rad/sec, $1/T_L$ decreases from 0.5 sec for one-axis tracking to 0.25 rad/sec for two-axis tracking. A decrease in $1/T_L$ accounts for the increase in phase shift at frequencies below 2 rad/sec.

The remaining subject, CP, (Fig. 28) lowered his K-axis amplitude ratio over the entire spectrum. Significant differences of 1 to 3 dB occurred between 1 and 4 rad/sec. Phase changes were generally less than 10 degrees and of little practical significance.

This subject showed somewhat more low-frequency lead than either of the others. Since the equalization was the same for both one- and two-axis tracking, a single approximation is presented in Fig. 28. The low-frequency parameters of this approximation are $1/T_L = .06$ and $1/T_{I'} = .33$ rad/sec. The remaining time constants are the same as for the other subjects.

The average K/s^2 describing functions are shown in Fig. 29. Two-axis tracking caused a decrease in amplitude ratio at all but two measurement frequencies. The amplitude ratio decreased between 1 and 2 dB over the frequency range of significant error power (1-4 rad/sec). Phase differences were generally less than 10 degrees. None of the gain or phase differences was statistically significant.

Since there was no change in equalization, a single analytical approximation has been derived for the pair of describing functions. A good approximation is obtained with a zero ($1/T_L$) at .25 rad/sec, a pole ($1/T_N$) at 4 rad/sec, and a time delay of .07 sec. The lag constants T_I and $T_{I'}$ are zero.

Our decision to attribute the pole at 4 rad/sec to the neuromuscular system is based on the work of Magdaleno (Ref. 16). He discusses a simplified model of the neuromuscular system which contains one forward network (muscle load dynamics) and one feedback network (combined effects of the muscle spindles and Golgi tendon organs). His model shows that the break frequency associated with T_N decreases as the feedback gain is reduced.

The human controller may adjust his neuromuscular feedback, and consequently his T_N, when the controlled element dynamics are varied. When tracking with dynamics of K, the human controller is in effect attempting to reproduce the input waveform. In order to maintain the necessary control over the position of the control stick, he will probably generate a high level of neuromuscular feedback and a relatively small T_N. When tracking with dynamics of K/s^2, the controller is less concerned with precise control of stick position and behaves somewhat as a bang-bang controller. In this situation, the controller may reduce the neuromuscular feedback in order to obtain a high forward gain with a consequent increase in T_N.

Measured describing functions for the individual subjects are shown in Figs. 30 to 32. From Fig. 30 we see that in the two-axis situation subject RL lowered his amplitude ratio from 1 to 2 dB over the frequency range of 1 to 4 rad/sec. The accompanying phase-shift differences were generally less than 15 degrees and not statistically significant. Figure 31 shows that there was no significant change in the describing function of subject EK. Subject CP (Fig. 32) lowered his amplitude ratio from 2 to 3 dB over the range of significant error power. The differences were statistically significant at a number of measurement frequencies. The accompanying phase differences, some of which were statistically significant, were on the order of 10 degrees and provided a larger phase margin for two-axis tracking.

3. Summary

Two-axis tracking caused an average increase in the normalized mean squared error of about 70% for the total task. The performance degradation was about 60% on the K/s^2 axis and almost 100% on the K axis. The performance degradation was reflected in the describing functions primarily by a lowering of the amplitude ratio over the frequency range of significant error power and in some cases by a change in equalization. Some of the amplitude ratio differences on the K axis were statistically significant and were on the average greater than the amplitude ratio differences on the K/s^2 axis.

There were noticeable differences among the subjects. Two of the subjects showed greater percent changes in the K-axis NMSE than in the K/s^2 axis. These same subjects also changed their equalization on the K axis by providing more low-frequency lead in the two-axis situation. The remaining subject behaved differently in that (1) he showed greater performance degradation on the K/s^2 axis than on the K axis and (2) he failed to change his equalization on either axis.

D. VARIABILITY OF RESULTS

One of our original hypotheses was that there would be one-axis two-axis differences in the variability of the results. That is, we expected the two-axis situation to produce a less consistent as well as degraded performance, compared to the one-axis performance. Quantitative assessments of the variability of the NMSE and the describing functions, based on standard deviations, are presented in this section.

1. Variability of the NMSE

The standard deviation of the NMSE scores divided by the mean of the scores is used as a measure of the variability. Normalization of the standard deviation in this way provided a measure that was relatively independent of task difficulty. One such measure was obtained from each subject-axis-bandwidth condition for one- and two-axis tracking. Each mean and standard deviation was based on nine NMSE scores.

Table 11 shows the average variability for each axis condition for each of the three experiments. Each entry of Table 11a is the average of nine measures of variability--one for each subject-bandwidth condition of Experiment 1. Two measurements were used to compute the averages in Table 11b, one per subject, for the first variation of Experiment 2. Three measures, one per subject, provided the averages shown in Table 11c, for the third experiment. No test was performed for the statistical significance of the one-axis two-axis differences.

Changes in the variability of the normalized mean squared error performance were inconsistent. The X-axis variability was less during two-axis tracking, whereas the opposite trend occurred on the Y-axis. The total-task variability changed less than 5 percent for Experiments 1 and 2 and increased 25% for Experiment 3. These small and inconsistent changes do not appear to be of practical significance.

2. Variability of the Describing Function

The system performance (i.e., the NMSE) depended most critically on the amplitude ratio of the describing function over the octave or two of the frequency scales just below the input bandwidth. This portion of the spectrum contained most of the error power. Consequently, standard deviations of the amplitude ratio measurements at 1, 2, and 4 rad/sec were used as indicators of describing function variability. These standard deviations were available only from Experiment 3--the only experiment in which we computed more than one describing function per subject per condition.

The average standard deviations and the one-axis two-axis differences are shown in Table 12. Each entry is the mean of nine standard deviations--one per subject per measurement frequency. Each of the component standard deviations is based on three measurements. Differences were small and of little practical significance. Two-axis tracking resulted in an increased variability of only 0.1 dB on each axis.

E. REMNANT DATA

A sampling of remnant data are presented for Experiments 1 and 3. Data were obtained from RL and CP, the two subjects who participated in both of these experiments. In order to avoid inaccuracies caused by recirculating noise, only remnant data obtained from input-to-error and input-to-stick measurements have been considered. These remnant measures are required for testing various models of the two-axis controller. In particular, a change in the portion of the NMSE not linearly correlated with the input would indicate that not all of the one-axis two-axis difference in NMSE can be accounted for by a one-axis two-axis difference in the describing function.*

The error remnant data for Experiment 1 (homogeneous control situation) are shown in Table 13a and for Experiment 3 (heterogeneous dynamics) in Table 13b. The normalized mean

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* Since the describing function is a linear approximation to the human controller's behavior, a change in the describing function will affect only that portion of the error that is correlated with the input.

squared remnant error ($NMSE_u$) and the normalized mean squared total error (NMSE) are given for one- and two-axis conditions, and the one-axis• two-axis differences are tabulated. The NMSE is defined as the mean squared value of the component of error not linearly correlated with the input signal, divided by the mean squared input deviation. Each value of NMSE shown in Table 13a is the average of four 100-sec measurements, one for each axis for each subject. Each NMSE entry is the average of the appropriate NMSE's shown in Table 5. The corresponding entries in Table 13b have been computed in the same way, except each average is based on two rather than four measurements, since the two axes are considered separately.

One-axis, two-axis differences between remnant measures ($NMSE_u$) were relatively small for the homogeneous control situation. Two-axis tracking produced an increase of 0.013 in the $NMSE_u$, which represents roughly a 10% increase. This increase accounted almost entirely for the increase in total NMSE.

A similar difference (0.015) in $NMSE_u$ occurred on the K axis when the dynamics were heterogeneous. This increase represented a more than tripling of the one-axis $NMSE_u$, but nevertheless accounted for less than half of the increase in the total NMSE. The increase in $NMSE_u$ was larger on the K/s^2 axis (0.047) and again accounted for about 60 percent of the total NMSE increase.

F. SUMMARY

Tracking in a homogeneous control situation produced one-axis two-axis performance differences that were, in general, statistically significant but small. There were no significant differences in total-task performance when the input bandwidths were heterogeneous. Performance differences that occurred on the individual axes disappeared when the input amplitudes were adjusted to produce approximately equal amplitudes on the two axes.

Relatively large changes occurred when the controlled element dynamics were heterogeneous. The normalized mean squared error increased an average of about 60% on the axis with acceleration dynamics and almost 100% on the axis with proportional control. The total-task NMSE increased about 70% in the two-axis situation. Much of the increase in NMSE was accounted for by a lower two-axis amplitude ratio over the range of frequencies in which the error power was significant. In addition, two of the subjects changed their equalization on the K axis by providing greater low-frequency lead during two-axis tracking.

The variability of the human controller's performance was essentially the same in the one- and two-axis control situations. There were relatively large one-axis two-axis differences in the uncorrelated portion of the normalized mean squared error when the dynamics were heterogeneous, which suggests that the observed mean squared error differences cannot be attributed entirely to changes in the describing function.

VI. DISCUSSION

A. FACTORS AFFECTING TWO AXIS PERFORMANCE

Of the many factors tested in this series of experiments, only three appeared to have had an important effect on the performance of the human controller in a two-axis task relative to that in a one-axis task. These are: (1) visual-motor interference effects; (2) attentional effects; and (3) the requirement of generating simultaneously two different equalizations. The last one of these had the greatest effect on system and human controller performance.

1. Visual-Motor Interference

When the human controller is controlling simultaneously in two axes and is using an integrated display and control, we would expect a certain amount of cross-coupling or interference between the stick movements intended for the two axes. This interference may be introduced in the visual system where the presence of an error or error velocity in one axis may interfere with the ability of the human controller to estimate error displacement and velocity in the other axis. The interference may also occur in the motor system where a movement intended for one axis inadvertently leads to a movement in the other. In addition, random cross-coupling introduced at the central processing level may produce interference effects similar to those introduced in the visual and motor systems. In our experiments we cannot distinguish among interference introduced at the visual, motor, or central processing level, and we shall lump all three types of interference together under the designation visual-motor interference.

It is reasonable to assume that the components of the error on one axis resulting from visual-motor interference effects will be uncorrelated with the input forcing function on that axis. This assumption can be justified by noting that the forcing function inputs on the two axes were uncorrelated in our experiments. Moreover, since we would expect that the human controller would attempt to compensate for any systematic (i.e., non-random) cross-coupling, the components of motion due to interference would be random at least with respect to their direction. Given the assumption that the interference components are uncorrelated with the input, we can consider the interference to be a source of remnant.

Table 13a shows that for the homogeneous control situation almost all of the increase in NMSE is accounted for by the increase in the $NMSE_u$, the uncorrelated portion of the normalized mean-squared error. In the case of Experiment 3, which was performed with heterogeneous dynamics, the increase in $NMSE_u$ accounted for almost half of the increase in NMSE. If we assume that the increase in $NMSE_u$ observed in going from one- to two-axis tracking is entirely due to visual-motor interference, we can account for almost all of the degradation in performance observed with homogeneous dynamics.

A simple model for predicting visual-motor interference effects is obtained by assuming that the interference component on an axis is proportional to the stick displacement on the other axis. Under this assumption, the component of error on axis A due to interference from axis B is

$$E_{IA} = k_{BA}{}^{2} \int_{o}^{\infty} S_{SB} \left| \frac{C_A}{1+H_A C_A} \right|^{2} d\omega \qquad (11)$$

where E_{IA} is the mean-squared error on axis A due to interference, S_{SB} is the power density spectrum of the stick movement on axis B, C_A is the transfer function of the controlled-element on axis A, H_A is the human controller's A-axis describing function, and k_{BA} is the interference coefficient.

Since

$$S_{SB} = S_{IB} \left| \frac{H_B}{1+H_B C_B} \right|^{2} \qquad (12)$$

where S_{IB} is the input power density on axis B, we can rewrite Eq. (11) as follows:

$$E_{IA} = k_{BA}{}^{2} \int_{o}^{\omega_i} \frac{S_{IB}}{|1+H_B C_B|^{2}} \left| \frac{H_B}{H_A} \right|^{2} \left| \frac{H_A C_A}{1+H_A C_A} \right|^{2} d\omega \qquad (13)$$

where ω_i is the cut-off frequency of the input.

For Experiment 1, in which the controlled-element dynamics in the two axes were the same, we can expect $H_B \doteq H_A$. Furthermore, in the region in which most of the input power is concentrated, the closed-loop transfer function in the A axis is approximately unity. Under these assumptions Eq. (13) reduces to

$$E_{IA} = k_{BA}^2 \int_0^{\omega_i} \frac{S_{IB}}{|1+H_B C_B|^2} \, d\omega \tag{14}$$

Therefore

$$NMSE_{IA} = k_{BA}^2 \times NMSE_B \tag{15}$$

where $NMSE_{IA}$ is the normalized mean-squared error due to visual-motor interference.

The data from Experiment 1 can be used to estimate the interference coefficient. We find from the data in Table 13a that $k_{BA}^2 = 0.05$. We can check this value of k_{BA}^2 by applying Eq. (13) to the results of Experiment 3 given in Table 13b. Note that in this case $H_A \neq H_B$. However, if we use the average values of H_A and H_B in the octave terminating at = 3.5 rad/sec, Eq. (13) reduces to

$$NMSE_{IA} \doteq k_{BA}^2 \left| \frac{H_B}{H_A} \right|^2 NMSE_B \tag{16}$$

Using the value for k_{BA}^2 determined above (.05), and letting H_A = 10 dB and H_B = 0 dB, we find that $NMSE_{IA} \doteq .001$ and $NMSE_{IB} \doteq .041$, where axes A and B refer to the K and K/s^2 axes, respectively. The corresponding changes in $NMSE_u$ are .015 and .047.

Thus, this simple model predicts well the change in $NMSE_u$ in the K/s^2 axis but not in the K axis. This result is not surprising since ${k_{AB}}^2$ was obtained using data from an axis in which the controlled-element was K/s^2. Apparently, the interference coefficient depends upon the dynamics of the axis in which the interference occurs.

2. Allocation of Attention

Allocation of attention is the extent to which a subject concentrates on or gives preference to one axis of the control task at the expense of the other when tracking two axes simultaneously. Attentional effects of this kind are evident in the results of all three of the experiments we performed. They are most clearly seen, however, in the behavior of subject RL in Experiment 2. As indicated in Table 8, when the mean-squared inputs in the two axes were equal, the NMSE of subject RL on the low-bandwidth axis showed a large increase in going from one to two axes, whereas the NMSE on the high-bandwidth axes actually decreased slightly. When controlling two axes simultaneously, the subject apparently attended more to the high-bandwidth axis, which was the source of most of the mean-squared error in the two-axis task, than he did to the low-bandwidth axis. This behavior was reasonable if the subject's objective was to minimize the total mean-squared error, and if by attending differentially to the more difficult task, he could actually improve his performance on that axis.

When the input amplitudes were adjusted so that the single-axis, mean-squared error on the two axes were approximately equal, the one-axis, two-axis performance differences were about the same for the high and the low bandwidth axes. In this situation each axis contributed about equally to the total mean-squared error and it was important for the subject to attend to each about equally.

This same kind of concentration on the axis contributing most to the total error is seen in the results of Experiment 3 in Table 9. When tracking two axes simultaneously with dissimilar controlled-element dynamics, subject EK managed to hold the increase total-task performance NMSE to less than 40 percent even though the K-axis NMSE increased over 100 percent. The NMSE in the K-axis was much smaller than in the K/s^2 axis, and minimization of the total mean squared error was more critically dependent on minimization of the errors on the K/s^2 axis. Much larger changes in human controller describing functions were observed on the K-axis for this and the other subjects than on the K/s^2-axis.

Some subjects showed a consistent preference for one axis over the other in the sense of apparently attending differentially to the preferred axis on the two-axis task. Subject CP consistently exhibited a preference for the X-axis. For example, his NMSE increased 20 percent on the Y-axis, but decreased 10 percent on the X-axis when he tracked the 3.5 rad/sec forcing functions in Experiment 1, (Table 6). This behavior was not dictated by the experimental conditions, since the actual (i.e., not normalized) mean-squared tracking error was slightly greater on the Y- than on the X-axis in the two-axis situation. His performance showed no significant change on either axis when he controlled simultaneously a high-bandwidth and a low-bandwidth input on the X-axis in Experiment 2. His natural preference for the X-axis input on the Y-axis apparently counterbalanced the nonhomogeneity of the experiment conditions. This subject also showed a smaller fractional increase in NMSE on the K-axis (X-axis) when tracking with heterogeneous dynamics, even though much larger errors were present on the K/s^2 axis.

We were not able to measure directly the allocation of attention in these experiments with integrated display and control. As a result it was not possible to determine quantitatively the relation between attention and performance. The attentional process that we encountered in these experiments is not a sampling process such as would be observed in experiments with non-integrated displays. Sampling would lead to an increase in effective time delay which is not seen in the describing functions we obtained. The attentional process that operates in the type of experiments we conducted appears to be more easily described in terms of an importance weighting associated with each of the components of the total-task error.

3. Heterogeneity of Required Equalization

The largest one-axis, two-axis performance differences were observed in Experiment 3 in which the controlled-element dynamics in the two axes were different. From Table 9 we see that the K-axis NMSE in the two-axis task was twice that in the single-axis task. The K/s^2-axis NMSE was 60 percent larger in the two-axis situation than in the one-axis situation. Large one-axis, two-axis differences in human controller describing functions were observed in the K axis of this experiment. The two-axis gain, lead break frequency, and gain cross-over frequency were generally lower in two-axis tracking than in one-axis tracking.

It is clear from the results of Experiment 3 and the other two experiments that the heterogeneity of the controlled-element dynamics is responsible for the large changes in performance and in human controller describing function. Experiment 1 demonstrated that two-axis operation was possible without important performance changes when the dynamics were homogeneous. Experiment 2 demonstrated that differences in spectral characteristics of the error signal and stick movements did not lead to important changes in total task performance. Table 13 shows that visual-motor interference effects can account for at most 40 percent of the increase in NMSE observed in Experiment 3. The source of the large changes in Experiment 3, particularly on the K-axis, is thus fairly well pinpointed to the heterogeneity of the controlled-element dynamics. This result is consistent with the results obtained by Verdi (Ref. 5) and Chernikoff (Ref. 7).

When the controlled-element dynamics in the two axes are different, the human controller must provide different kinds of equalization in the two axes in order to stabilize the system and to achieve good performance. For $C(\omega) = K$, lag equalization is required. In one-axis tracking, the amplitude ratio of the describing function of all three subjects exhibits a -20 dB/decade slope in the region of gain cross-over frequency. For K/s^2 tracking, the controller must generate a lead in order to stabilize the system and all three subjects showed such a lead.

It appears that the necessity for generating simultaneously two very different kinds of equalization is the cause of the large performance differences observed in Experiment 3. In the two-axis situation, the describing function for the K-axis changes so as to resemble more closely the K/s^2 axis describing function. Table 10 shows that subject RL adopted a lead-lag characteristic in the K-axis in the two-axis situation. Subject EK decreased the lead break frequency, thereby producing a more important lead-lag effect in two-axis tracking. Subject CP did not show important changes in lead-lag frequencies, but he already had an important lead-lag in single axis tracking. Moreover, the one-axis, two-axis differences in his K-axis NMSE were the smallest of the three subjects.

This conclusion is weakened by the fact that two of the three subjects show evidence of a low-frequency lead-lag in one-axis K tracking. However, for one of these subjects (EK) the separation between the lead and lag break frequencies was only one octave and the lead-lag had a relatively small effect on the describing function in one-axis tracking. For the other of these two subjects (CP) the lead-lag separation was about 2-1/2 octaves and had an important effect on the describing function at very low frequencies. However, this subject showed only minor one-axis, two-axis differences in the parameters of his describing function.

The important lead-lag observed in the one-axis K-axis describing function of subject CP and to a lesser extent with subject EK was unexpected. Such an effect was not found by Elkind (Ref. 1) in his study of K-axis tracking. One possible explanation for the presence of this effect in the describing functions of subject CP is that he was trained first with K/s^2 dynamics and had extensive practice with this type of controlled-element before they started training with K dynamics. Moreover, much of the K training for both subjects was in a two-axis situation in which the other axis had a K/s^2 controlled-element. They may have simply continued the strategy of providing a lead-lag that was appropriate to K/s^2 tracking when controlling K dynamics. Figure 10 of Ref. 11, which contains describing functions obtained in our preliminary experiments, shows that the describing function of subject EK did not exhibit a low-frequency lead-lag before that subject was trained with heterogeneous dynamics.

It should be noted that the large one-axis, two-axis differences observed in Experiment 3 were obtained after all subjects had received extensive training in this control situation. Subjects CP and RL had about 6 hours of experience in this situation with heterogeneous dynamics. Subject EK had 8 hours. There is no indication that further training would have eliminated or reduced the differences in performance materially.

B. FACTORS NOT AFFECTING TWO-AXIS PERFORMANCE

Many of the factors tested in our experiments do not appear to have an important effect on the performance of the human controller in a two-axis control situation. These are: (1) information transmission rate limitations, (2) switching mechanisms, (3) single-channel central processing, and (4) increased variability in two-axis control situations.

1. Information Processing Limitations

The rate at which information is transmitted in a continuous tracking task can be defined (Ref. 17) as

$$R \doteq \int_{o}^{W_i} \log_2 \frac{S_i(f)}{S_N(f)}\, df \tag{17}$$

$$R \doteq W_i \log_2 (NMSE_u)$$

where W_i is the input bandwidth in cycles/second and $S_i(f)$ and $S_N(f)$ are the power density spectra of the input and the remnant. Estimates of the information rate for Experiments 1 and 2 are given in Table 14. There were made using Eq. (17) and the data from Table 13. For Experiment 1, the total information rate in the two-axis situation was very nearly twice that of the one-axis situation. Clearly, the human controller was not information rate limited in this experiment. In the K-axis of Experiment 3, the single-axis information rate of 4.1 bits/sec was about equal to the information rate obtained by Elkind and Sprague with a similar input in their studies of information transmission with single-axis compensatory systems in which the dynamics were equal to K (Ref. 17). In the two-axis task the information rate in the K-axis dropped to 3.1 bits/sec. The total information rate of 5 bits/sec for the two-axis heterogeneous situation was less than the sum of the information rates obtained in each axis separately.

We do not think that this decrease in information rate is evidence for an information rate limit. First of all, information rates as high as 8 bits/sec have been observed in continuous tracking (Ref. 7). Second, in preliminary experiments with subject EK, in which we compared one- and two-axis tracking with K dynamics on both axes, we found that the NMSE in each axis did not increase in going from one to two axes. We infer from this result that the information rate per axis remained about constant and that the controller transmitted twice as much information in the two-axis task as he did in the one-axis task.

2. Switching Mechanisms

The human controller could exhibit switching behavior at the sensory, central processing, or motor levels when performing a multi-dimensional task. Switching would result if he attended sequentially to the several components of the multi-dimensional task. If non-integrated displays are employed, switching behavior would be necessary as the subject scanned the several displays. In our experiments with an integrated display such scanning was not necessary.

However, the subject could still monitor and correct errors on X and Y alternately. Errors on one axis would tend to increase while errors on the other axis were being reduced. At the output level, corrective motions would tend to appear on one axis at a time. Additional switching would occur at the central processing level in the heterogeneous dynamics task if the subject were able to generate only one type of describing function at a time. Switching at this level should also appear at the motor level, since the subject could not respond until a strategy has been formulated.

Any type of switching behavior in a two-axis tracking situation that is not present in a one-axis situation should increase the human controller's effective time delay and thereby increase his phase lag at high frequencies. We found no consistent increase in phase lag at high frequencies with the addition of a second axis. Furthermore, visual inspection of the recordings of error and control waveforms did not reveal the effects of switching. Thus, we conclude that switching mechanisms are not an important factor in two-axis tracking with integrated display.

3. Single-Channel Central Processing

The fact that the human controller can perform about as well in each axis of a two-axis control situation with homogeneous dynamics as he could in a single-axis situation implies that he can process both channels of information simultaneously. Furthermore, since there is no evidence for a switching mechanism it appears that the two channels are processed in parallel rather than sequentially.

In the case of Experiment 3 (heterogeneous dynamics) we found large one-axis, two-axis differences in human controller describing functions. It appears that the requirement for generating two different equalizer characteristics imposes some strain on the controller's signal processing abilities. Although apparently he still processes both channels in parallel, the two-axis describing functions tend to resemble each other more closely than do the one-axis describing function.

4. Short-Term Variability

We originally expected that the addition of a second axis would increase the variability of the human controller's characteristics, especially in the subjectively complex task of tracking with heterogeneous dynamics. Contrary to our expectations we found that the one-axis, two-axis differences in variability of NMSE and describing functions were relatively small and not significant. Thus, there is no evidence for increased variability in two-axis tracking and this factor can be eliminated as a possible source of degradation of the human controller's performance.

C. MODIFICATION OF THE SINGLE-AXIS MODELS

The single-axis human controller models provide predictions of two-axis behavior that are reasonably good in control situations in which the controlled-element dynamics are homogeneous. These predictions will, however, be subject to errors from two sources--visual-motor interference and differential attention. The single-axis models should be modified to include these factors if accurate predictions are to be achieved.

The amount of visual-motor interference will depend upon the controlled-element dynamics and upon the design of the control device. The simple proportional cross-coupling model proposed in this section seems to provide a reasonably good prediction of interference effects once the coupling coefficient is determined. The coupling coefficient, however, will have to be determined in a separate experiment.

Differential allocation of attention can have an important affect on system performance in an axis which is receiving little attention. The allocation of attention appears to depend upon subject training and preferences, signal amplitude, and no doubt upon other factors such as display sensitivity and instructions. We do not yet know how to measure attention or to predict its effect on performance. A good procedure to follow in experiments with two-axis systems would be to attempt to control for attentional effects to insure that equal attention is allocated to each axis.

The most important changes to the one-axis models are required when the controlled-element dynamics are heterogeneous. In this situation major changes in the form of the human controller's describing function are observed. It appears that when different equalizer characteristics are required of the subject for the two axes, his two-axis describing functions tend to resemble each other more than they do in a one-axis tracking situation. We have not yet been able to develop methods for predicting the extent of the changes in describing functions that occur in this kind of heterogeneous situation. This is one of the important problems in manual control that requires solution.

The factors that we find to be important in two-axis control situations are likely to be important also in higher dimensional control situations. We have not done any experiments with such higher dimensional systems, and therefore cannot predict whether these effects are greater or less in such situations. This problem will be investigated in future experiments.

VII. CONCLUSIONS

Three factors that affect human controller characteristics in two-axis control situations have been identified. These are: (1) visual-motor interaction, (2) differential allocation of attention, and (3) nonhomogeneity of required equalization when the controlled dynamics are not homogeneous. Single-axis describing function models for the human controller must be modified to include the effects of these factors in order to obtain accurate predictions of human controller characteristics in two-axis situations. These factors are also likely to have important effects on performance in higher-dimensional control situations.

A simple model has been developed for predicting visual-motor interference effects. Models for the prediction of attention and equalization effects have not yet been developed. Further work is required to develop such models and to determine how to apply them to higher dimensional systems.

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TABLE 1

Controlled-Element Dynamics for Experiment 3

Subject	Controlled-Element Dynamics	
	X Axis	Y Axis
RL	4	$64/s^2$
CP	4	$64/s^2$
EK	$64/s^2$	4

TABLE 2

Participation of Subjects

Subject	Experiment 1 Homogeneous Control Situation	Experiment 2 Heterogeneous Inputs	Experiment 3 Heterogeneous Dynamics
RL	x	x	x
CP	x	x	x
BL	x		
EK*			x

* Data from this subject were obtained during the preliminary phases of the experimental program.

TABLE 3

Formal Experimental Plan

Session	Run Number	Task
1	1	X-AXIS
1	2	Y-AXIS
1	3	2-AXIS
2	4	Y-AXIS
2	5	2-AXIS
2	6	X-AXIS
3	7	2-AXIS
3	8	X-AXIS
3	9	Y-AXIS

TABLE 4

Experimental Plan for Subject E.K.

Session	Run Number	Task
1	1-3	2-axis
2	4-6	X-axis
3	7-9	Y-axis
4	10-12	X-axis
5	13-15	Y-axis
6	16-18	2-axis
7	19-21	Y-axis
8	22-24	2-axis
9	25-27	X-axis

TABLE 5

Performance Levels for Experiment 1
(Homogeneous Control Situation)

Normalized Mean Squared Error

	X axis		Y axis		Total Task	
Subject	1-axis	2-axis	1-axis	2-axis	1-axis	2-axis
	Input Bandwidth = 3.5 radians per second					
RL	0.22	0.24	0.24	0.27	0.23	0.26
CP	0.26	0.23	0.21	0.26	0.24	0.25
BL	0.24	0.27	0.20	0.27	0.22	0.27
3 subj	0.24	0.25	0.22	0.27	0.23	0.26
	Input Bandwidth = 2.5 radians per second					
RL	0.11	0.11	0.08	0.09	0.10	0.10
CP	0.12	0.11	0.08	0.10	0.10	0.11
BL	0.17	0.16	0.10	0.13	0.13	0.14
3 subj	0.13	0.13	0.09	0.10	0.11	0.12
	Input Bandwidth = 1.5 radians per second					
RL	0.032	0.032	0.025	0.029	0.029	0.030
CP	0.051	0.045	0.026	0.035	0.039	0.040
BL	0.061	0.067	0.030	0.043	0.046	0.054
3 subj	0.048	0.048	0.027	0.036	0.038	0.042

TABLE 6

Differential Performance Levels for Experiment 1
(Homogeneous Control Situation)

Percent Change in Normalized Mean Squared Error

Input Bandwidth	Subject	X Axis	Y Axis	Total Task
	RL	9	13	12
3.5	CP	-10	20*	4
	BL	12	35**	22*
	3 subj	4	23***	13**
	RL	1	3	1
2.5	CP	-8	25**	5
	BL	-8	30**	6
	3 subj	5	20	4
	RL	0	13	5
1.5	CP	-12	34**	3
	BL	8	43*	19**
	3 subj	-1	30***	10*

* Significant at the .05 level
** Significant at the .01 level
*** Significant at the .001 level

TABLE 7

Differential Performance Levels for Experiment 1
(Homogeneous Control Situation)

Percent Change in Normalized Mean Squared Error,
Averaged over the three bandwidth conditions

Subject	X Axis	Y Axis	Total Task
RL	3	10	6
CP	-10	27	4
BL	4	36	16
3 subj	-1	24	9

TABLE 8

Performance Levels for Experiment 2
(Heterogeneous Inputs, Homogeneous Dynamics)

a. Normalized Mean Squared Error

Subject	X Axis (w_i=1.5)		Y Axis (w_i=3.5)		Total Task	
	1-axis	2-axis	1-axis	2-axis	1-axis	2-axis
CP	0.031	0.034	0.13	0.13	0.079	0.082
RL(1)	0.017	0.034	0.12	0.10	0.067	0.066
RL(2)	0.022	0.024	0.15	0.14	0.040	0.041

b. Percent Change in Normalized Mean Squared Error

Subject	X-axis (w_i=1.5)	Y-axis (w_i=3.5)	Total Task
CP	9	- 2	4
RL(1)	96***	-16**	-2
RL(2)	8	- 5	1

* Significant at the .05 level
** Significant at the .01 level
*** Significant at the .001 level

The mean squared input was 4.0 cm^2 for the X and Y axis conditions for subjects CP and RL(1). Mean squared X and Y inputs were 6.9 and 1.1 cm^2 respectively for subject RL(2).

TABLE 9

Performance Levels for Experiment 3
(Heterogeneous Dynamics, Homogeneous Inputs)

a. Normalized Mean Squared Error

Subject	K Axis		K/s^2 Axis		Total Task	
	1-Axis	2-Axis	1-Axis	2-Axis	1-Axis	2-Axis
RL	0.049	0.103	0.14	0.25	0.094	0.18
CP	0.039	0.062	0.11	0.22	0.075	0.14
FK	0.037	0.083	0.13	0.14	0.082	0.11
3 subj	0.042	0.083	0.13	0.21	0.083	0.14

b. Percent Change in Normalized Mean Squared Error

Subject	K Axis	K/s^2 Axis	Total Task
RL	110***	79***	87***
CP	59***	91***	81***
FK	124***	13*	38***
3 subj	98***	62	69

* Significant at the .05 level
** Significant at the .01 level
*** Significant at the .001 level

TABLE 10

Analytic Approximations to the Describing Functions of Experiment 3
(Heterogeneous Dynamics, Homogeneous Inputs)

		3 Subjects K-Axis		RL K-Axis		EK K-Axis		CP K-Axis	3 Subjects K/s^2 Axis
Parameter		1-Axis	2-Axis	1-Axis	2-Axis	1-Axis	2-Axis	1+2 Axes	1+2 Axes
τ	seconds	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.07
$1/T_N$	rad/sec	16	16	16	16	16	16	16	4
K_h	dB	9	1.5	13	2	10	2	0	-18
$1/T_L$	rad/sec	.25	.25	0	.33	.5	.25	.06	4
$1/T_{I'}$	rad/sec	.5	1	0	1	1	1	.33	0
$1/T_I$	rad/sec	2.5	2.5	2.5	2.5	2.5	2.5	2.5	0

The analytic approximations were of the form:

$$K_h \frac{(T_L s+1)\, e^{-\tau s}}{(T_{I'} s+1)(T_I s+1)(T_N s+1)}$$

TABLE 11

Variability of the Normalized Mean Squared Error

a. Homogeneous Control Situation

Axis	1-axis	2-axis	Percent Change
X	.062	.054	-12
Y	.066	.068	1
Total Task	.051	.050	- 2

b. Homogeneous Dynamics, Heterogeneous Inputs

X	.067	.058	-13
Y	.040	.049	22
Total Task	.037	.038	3

c. Heterogeneous Dynamics, Homogeneous Inputs

X	.054	.045	-17
Y	.046	.065	41
Total Task	.036	.045	25

TABLE 12

Variability of the Describing Function

Data were obtained from Experiment 3
(Heterogeneous Dynamics, Homogeneous Inputs)

Average Standard Deviation in dB

Axis	1-axis	2-axis	Difference
K	0.9	1.0	0.1
K/s^2	0.4	0.5	0.1

TABLE 13

Error Remnant Data

NMSE	total mean squared error
$NMSE_u$	mean squared error not linearly correlated with the input, divided by the mean squared input.

Axis	Number of Axes	$NMSE_u$	NMSE
a. Homogeneous Control Situation			
Average	1	0.096	0.234
of	2	0.109	0.250
X and Y	2 - 1	0.013	0.016
b. Heterogeneous Dynamics, Homogeneous Inputs			
K	1	0.006	0.044
	2	0.021	0.083
	2 - 1	0.015	0.039
K/s^2	1	0.048	0.13
	2	0.095	0.21
	2 - 1	0.047	0.08

TABLE 14

Estimates of Information Rate

Experiment	Axis	No. of Axes	R/Axis bits/sec	Total R bits/sec
1	Average of X and Y	1	1.9	1.9
1	Average of X and Y	2	1.8	3.6
3	K	1	4.1	4.1
3	K/s^2	1	2.4	2.4
3	K	2	3.1	5.0
3	K/s^2	2	1.9	

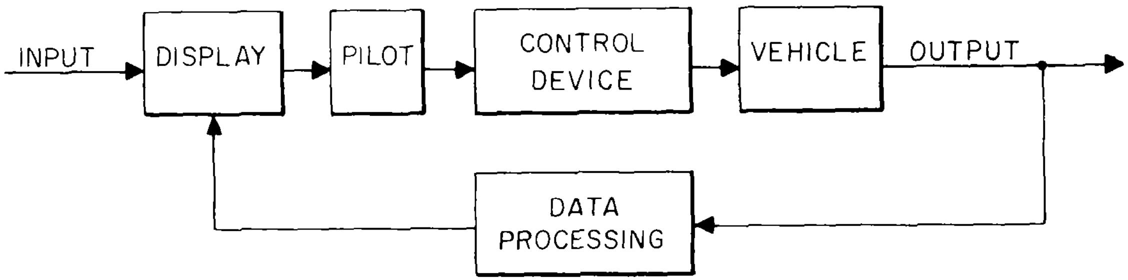


FIG. 1 BLOCK DIAGRAM OF SINGLE AXIS MANNED FLIGHT CONTROL SYSTEM

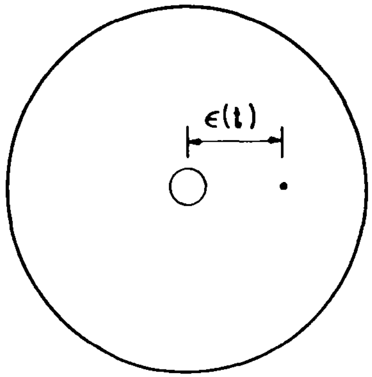


FIG. 2 SINGLE AXIS COMPENSATORY DISPLAY

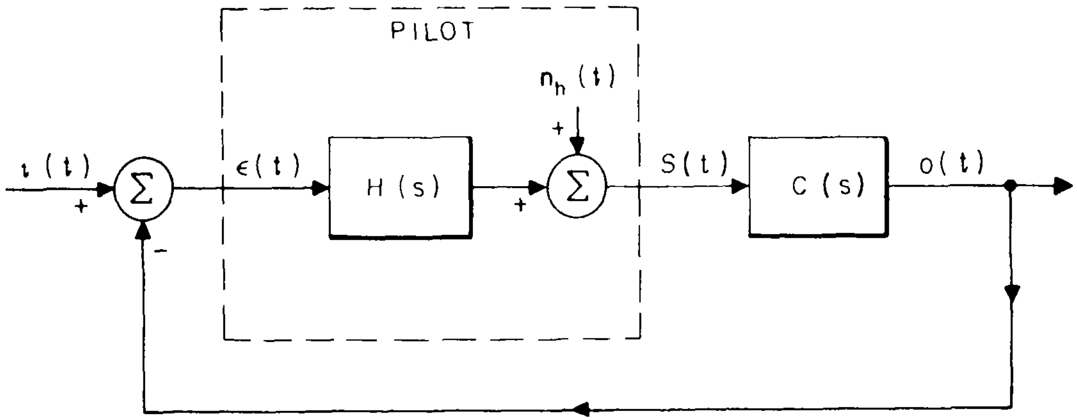


FIG. 3 SIMPLIFIED BLOCK DIAGRAM OF FLIGHT CONTROL SYSTEM

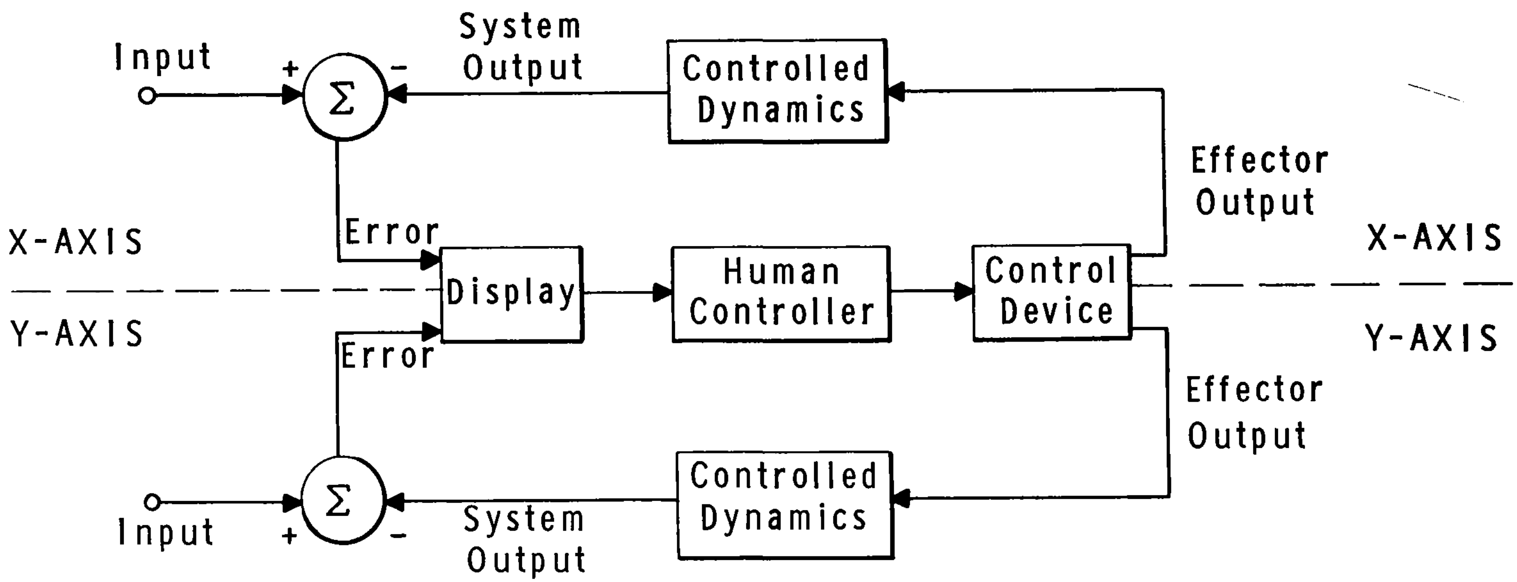


FIG. 4 BLOCK DIAGRAM OF A TWO-AXIS MANUAL CONTROL SYSTEM WITH INTEGRATED CONTROL AND DISPLAY CONFIGURATION

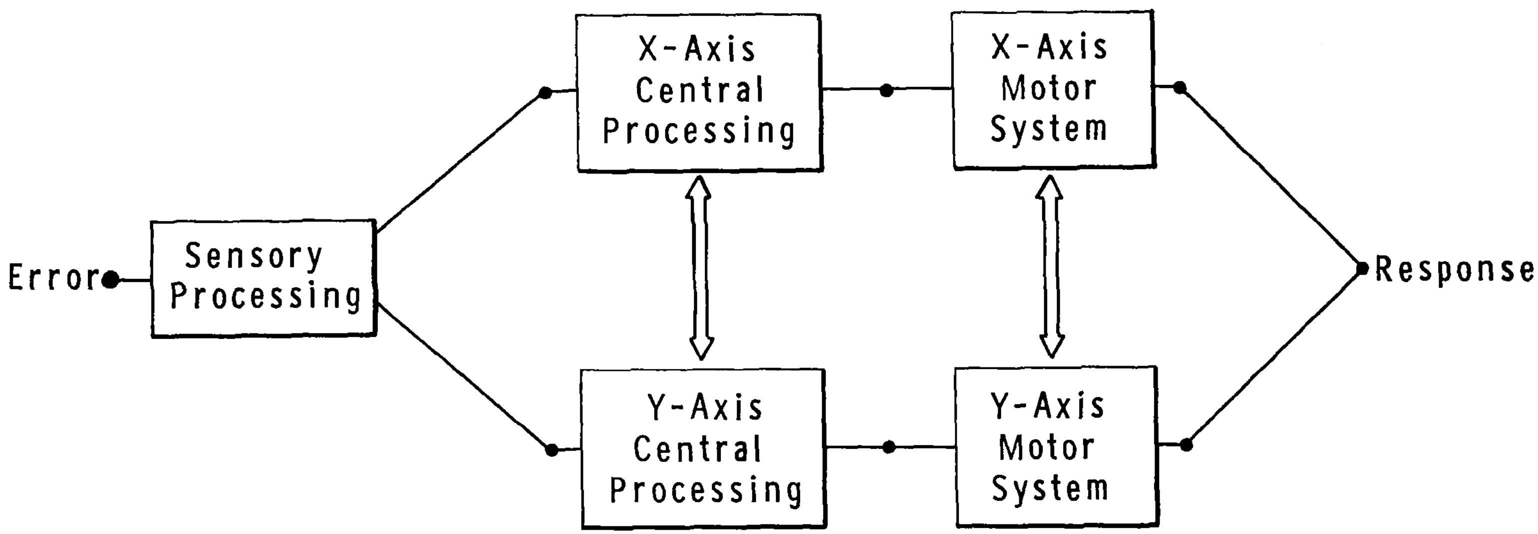


FIG. 5 BLOCK DIAGRAM OF THE HUMAN CONTROLLER WHEN PRESENTED WITH INTEGRATED TWO-DIMENSIONAL CONTROL AND DISPLAY CONFIGURATION

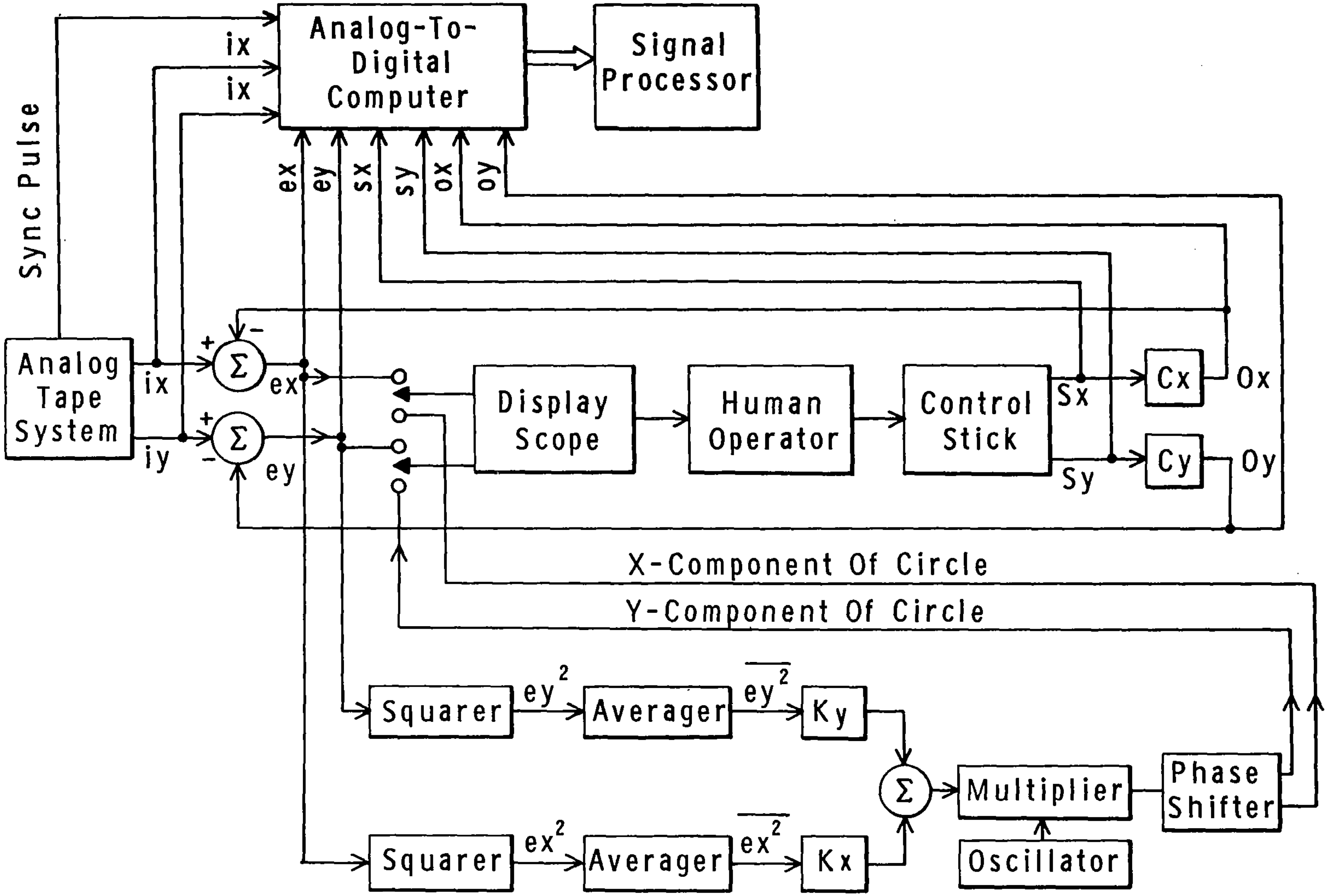
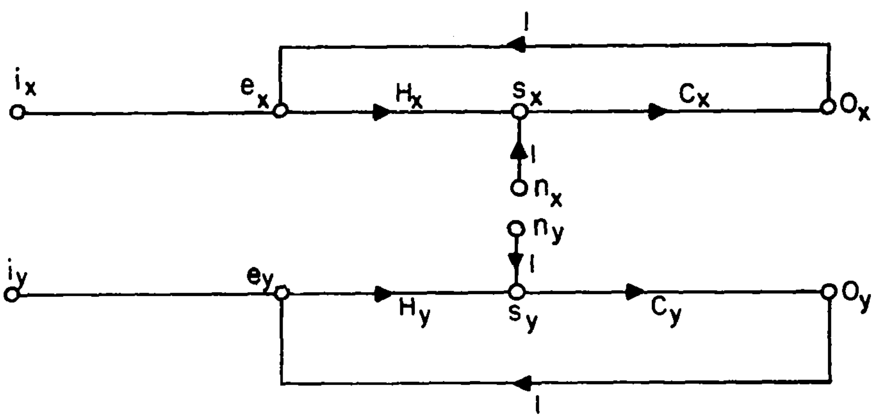


FIG. 6 FUNCTIONAL DIAGRAM OF THE TWO-AXIS TRACKING SYSTEM



i_x = x-component of the input forcing function.

e_x = x-component of the error displayed to the human operator.

s_x = x-component of the stick deflection(operator's response).

o_x = x-component of the system output.

n_x = x-component of the operator's response that is not linearly correlated with i_x or i_y.

H_x = linear relation between x-component of operator's response and x-component of error.

C_x = controlled element relating x-component of system output to x-component of stick deflection.

Signals and system functions in the Y axis correspond to those defined above for the X axis.

FIG. 7 LINEAR FLOW DIAGRAM OF THE TWO-AXIS COMPENSATORY TRACKING SYSTEM

FIG.8 SUBJECT BOOTH

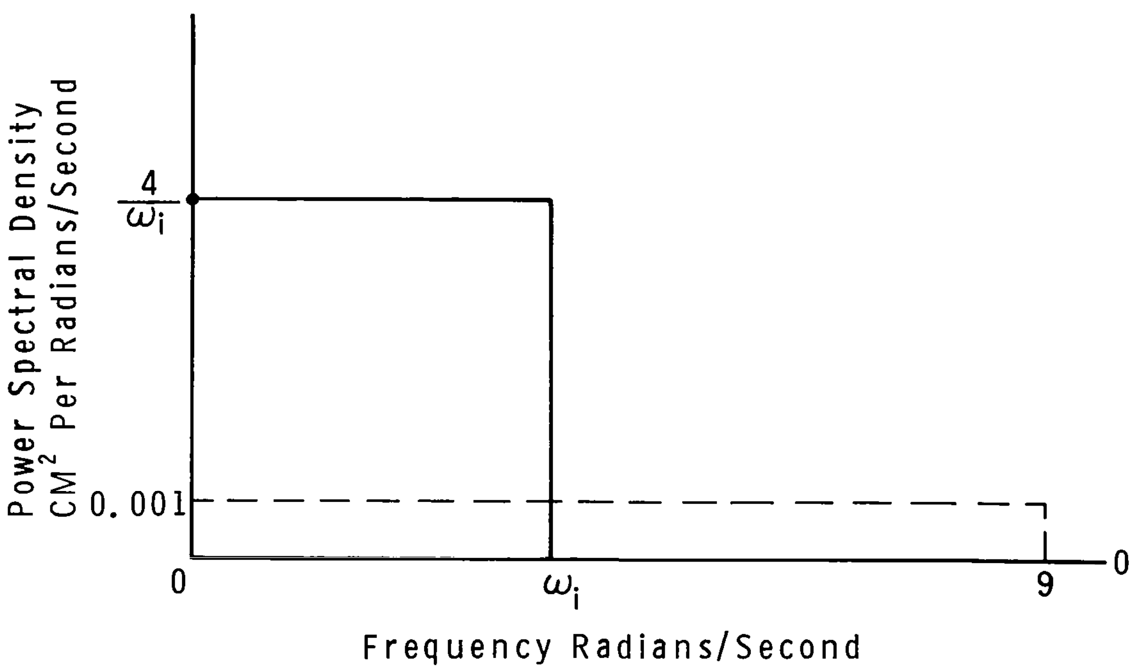


FIG. 9 THE POWER SPECTRAL DENSITY OF A FORCING FUNCTION HAVING A NOMINAL BANDWIDTH OF ω_i RADIANS PER SECOND

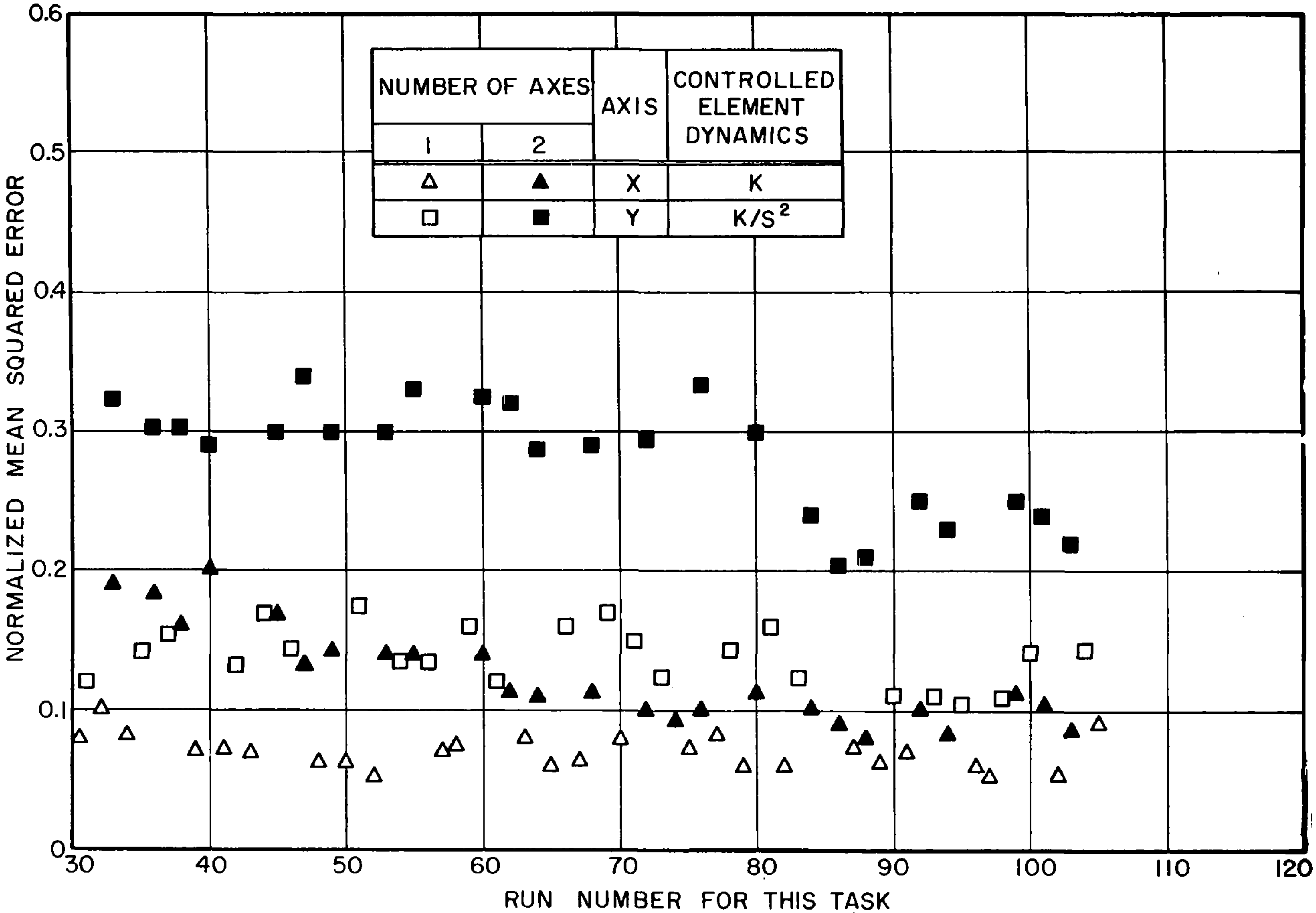


FIG. 10 TRAINING RECORD FOR SUBJECT RL FOR EXPERIMENT 3

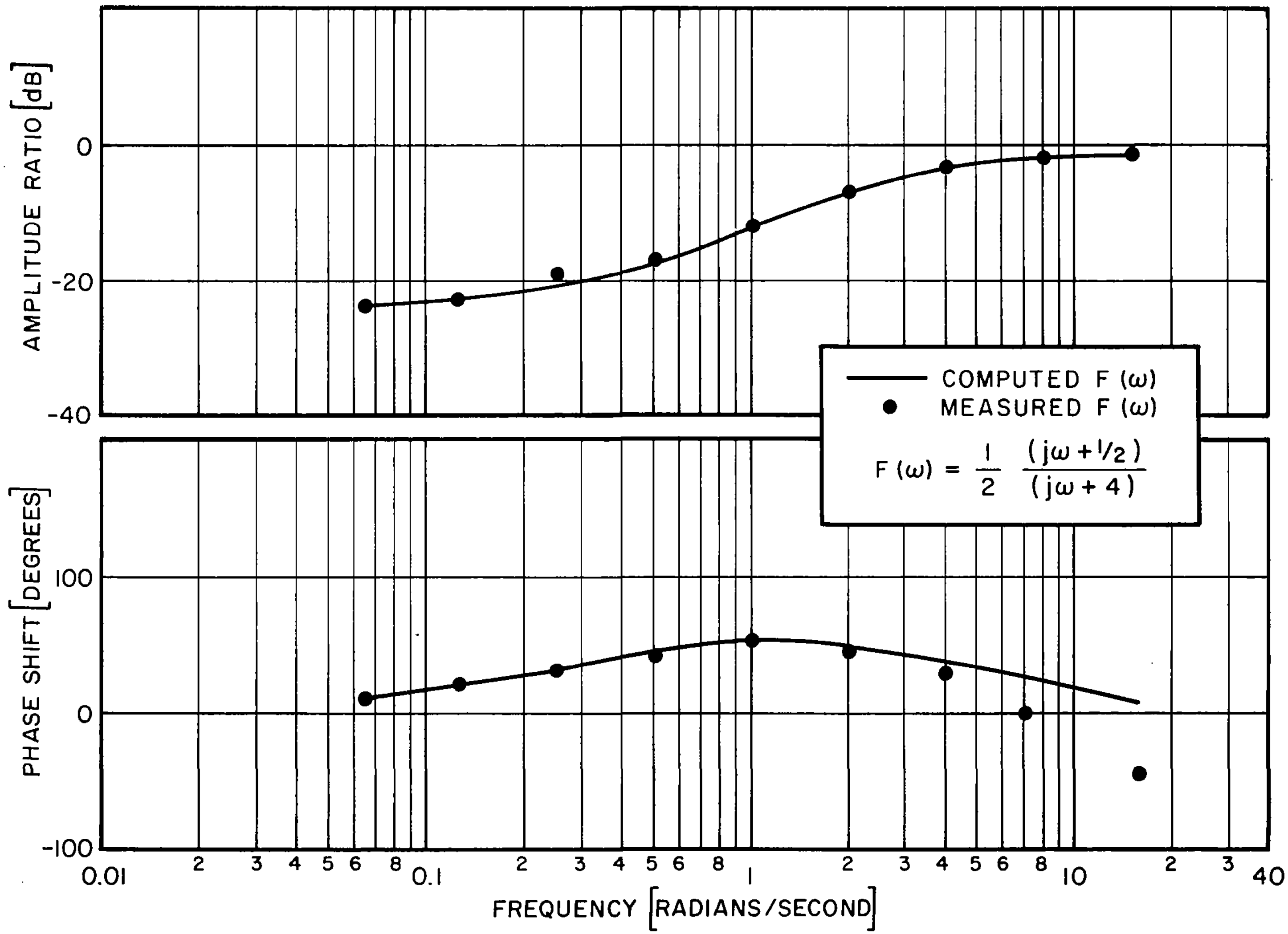


FIG. 11 OPEN-LOOP CALIBRATION TEST
Measured and Computed Transfer Functions of Test Filter

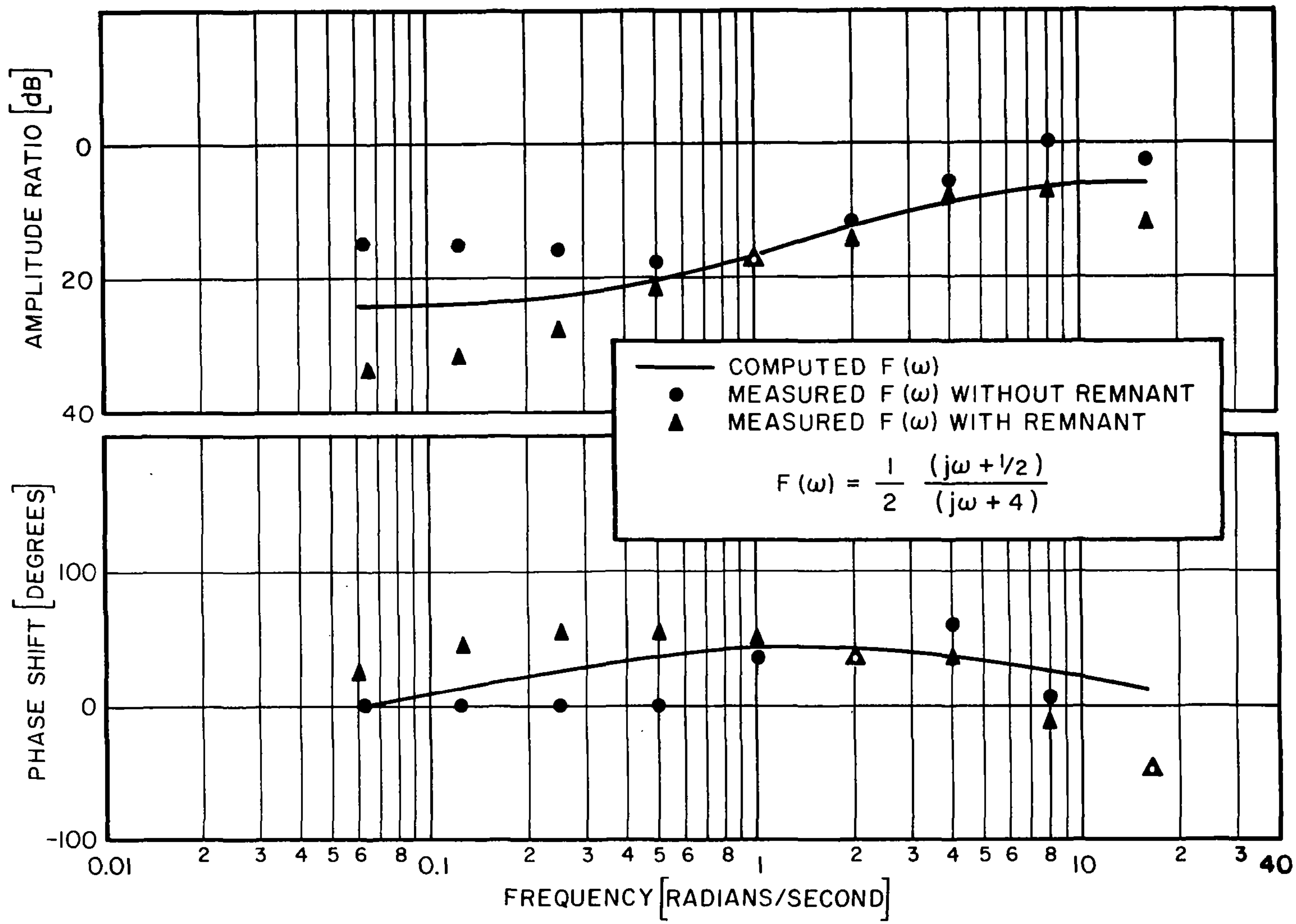


FIG. 12 CLOSED LOOP CALIBRATION TEST
Measured and Computed Transfer Functions of Test Filter

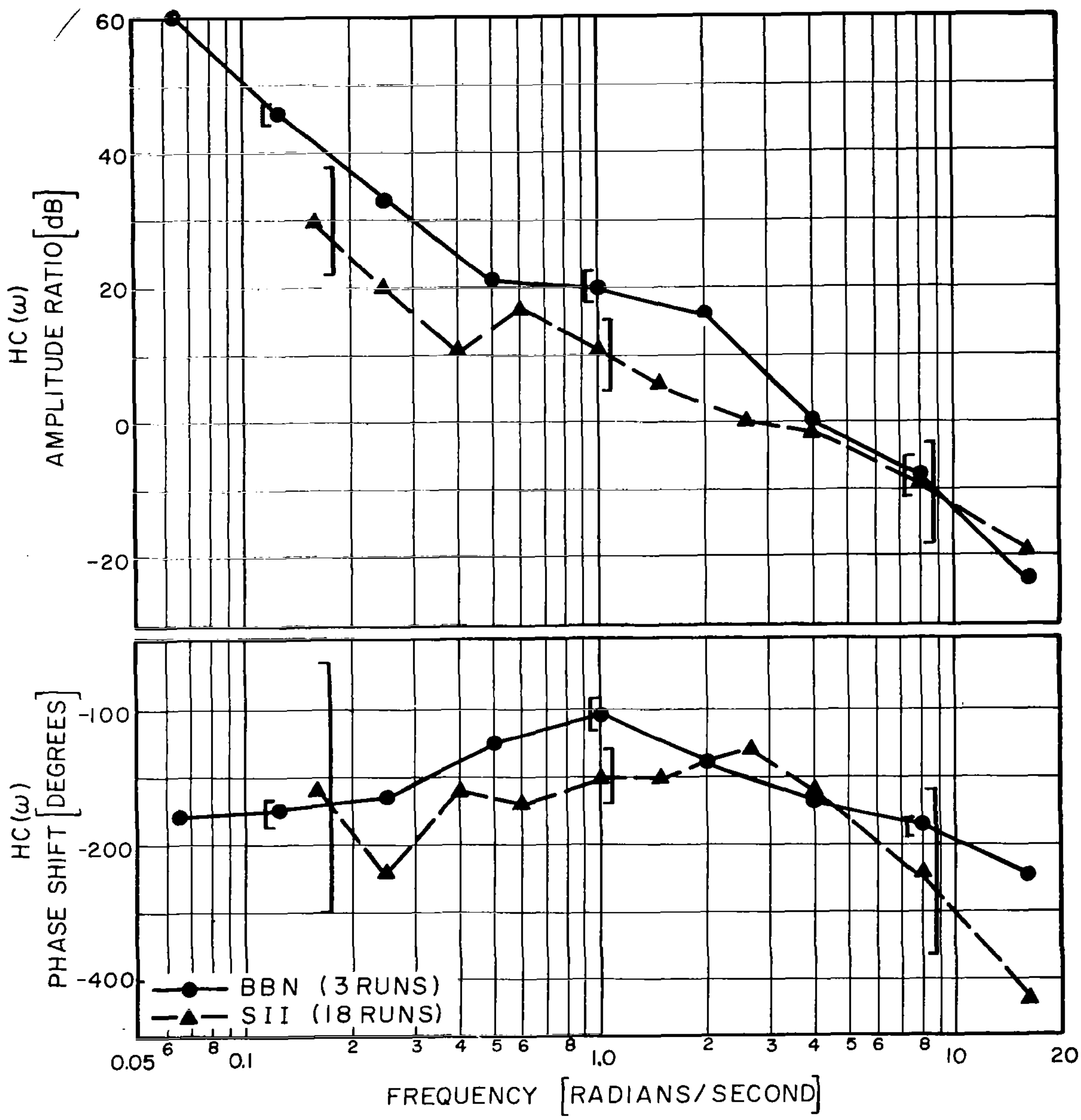


FIG. 13 COMPARISON OF OPEN LOOP DESCRIBING FUNCTIONS OBTAINED BY BBN AND STI

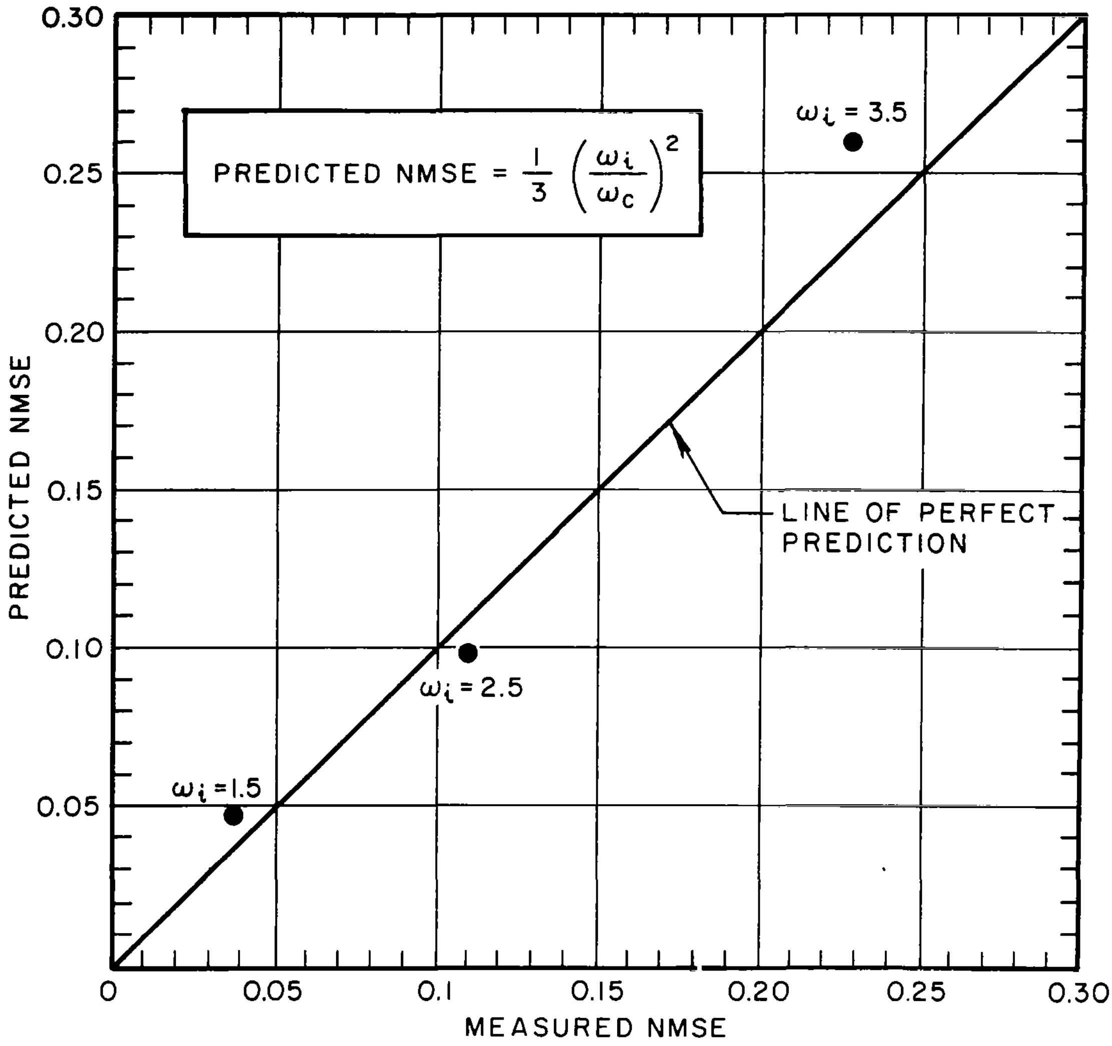


FIG. 14 COMPARISON OF NMSE PREDICTED BY ONE-THIRD LAW WITH MEASURED NMSE

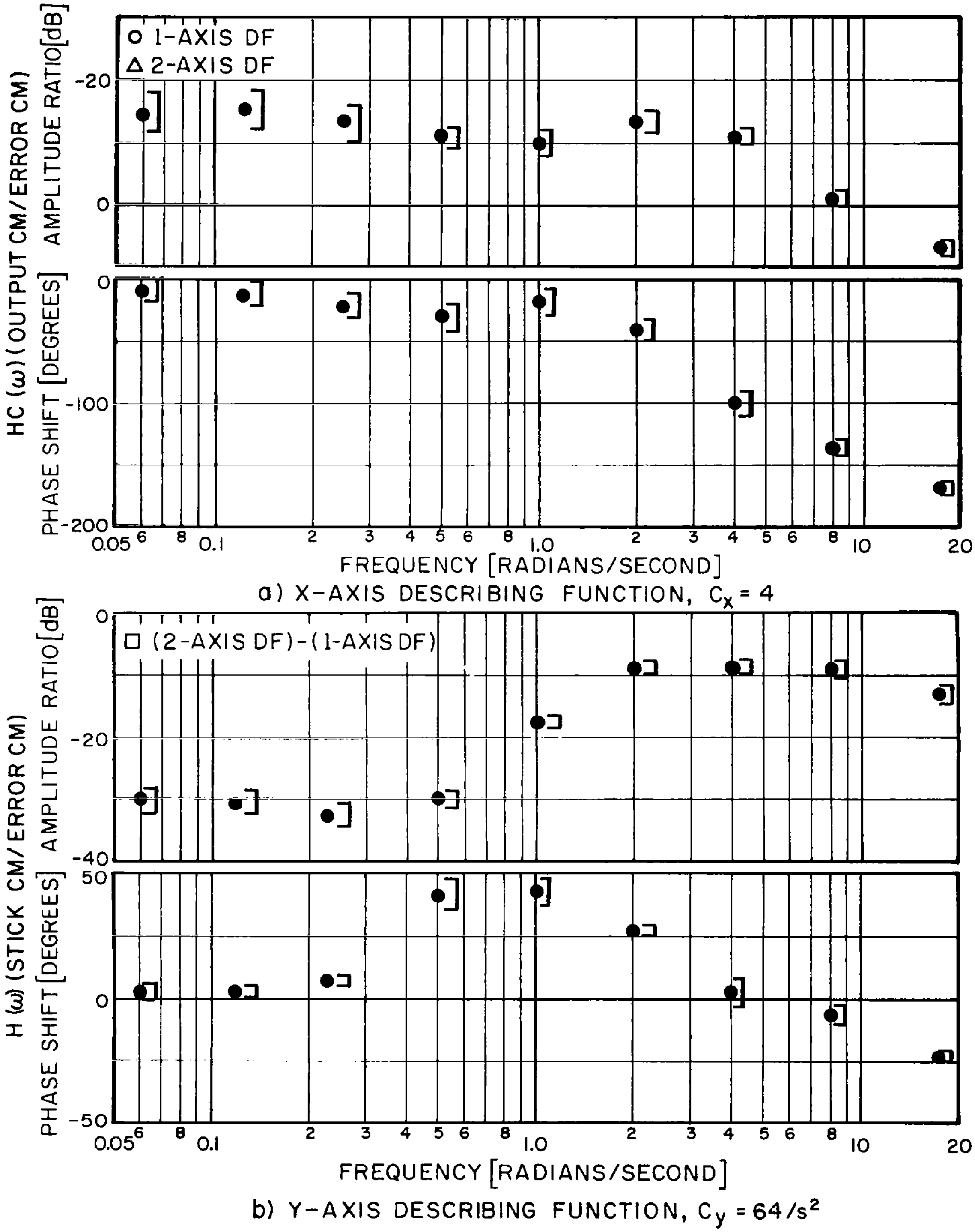


FIG. 15 INTRASUBJECT DESCRIBING FUNCTION VARIABILITY
Experiment 3 Heterogeneous Dynamics
Subject RL (3 Runs)

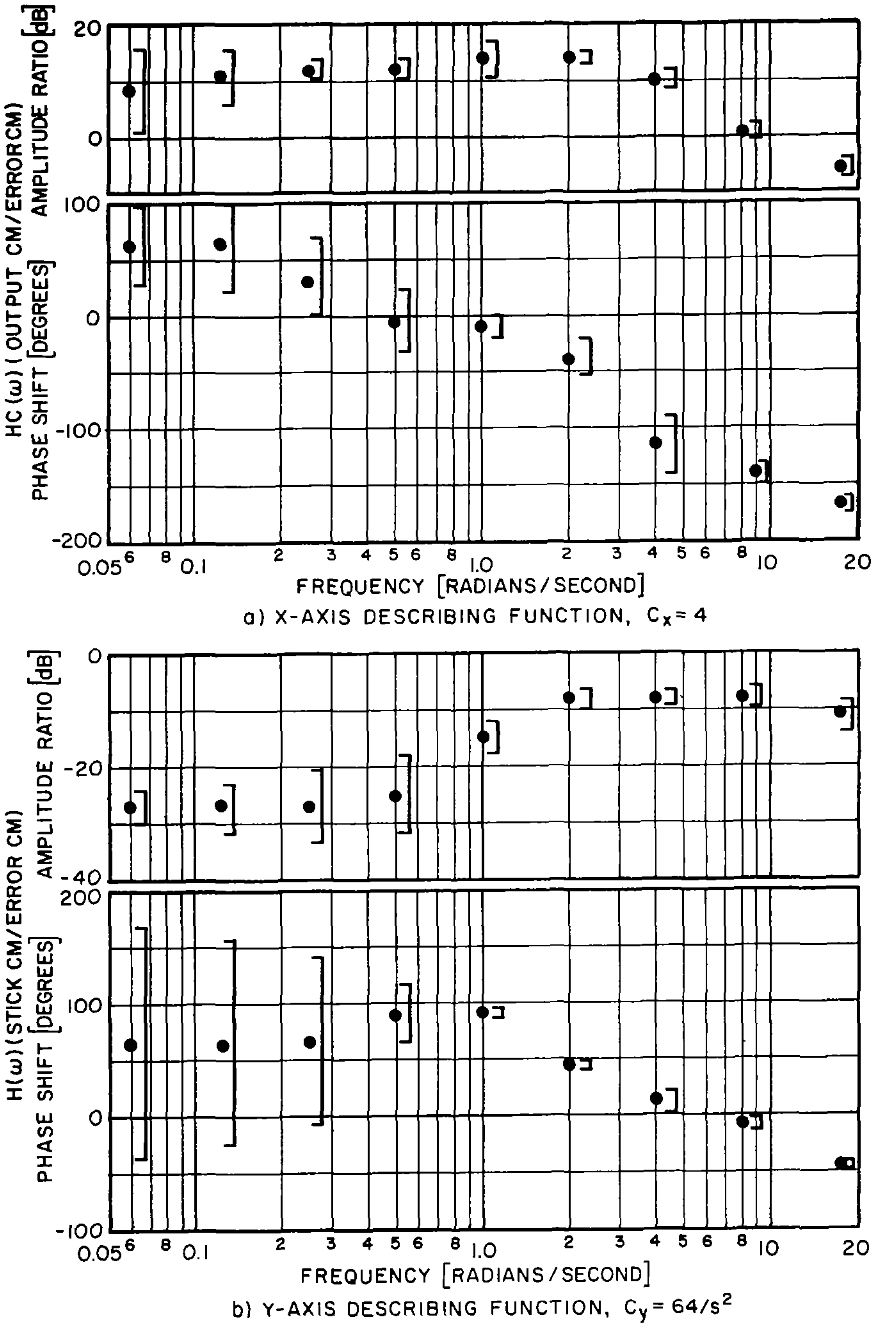


FIG. 16 INTERSUBJECT DESCRIBING FUNCTION VARIABILITY
Experiment 3 Heterogeneous Dynamics
Three Subjects (9 Runs)

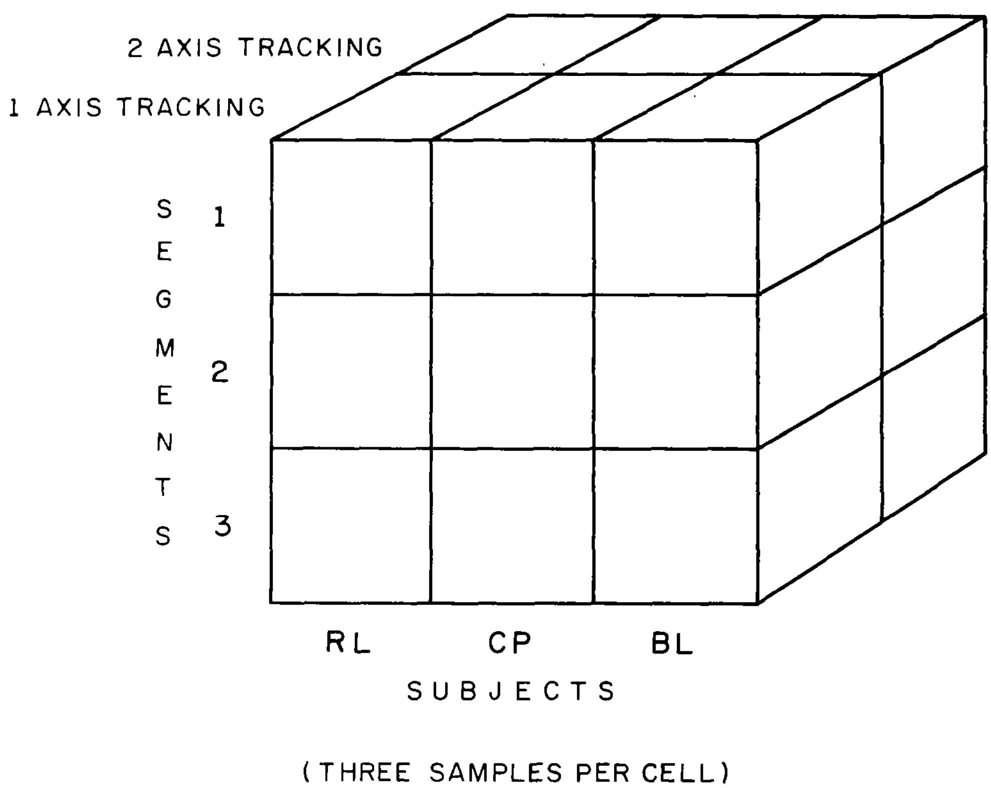


FIG. 17 FACTORIAL REPRESENTATIONAL OF DATA FOR EACH AXIS-BANDWIDTH CONDITION

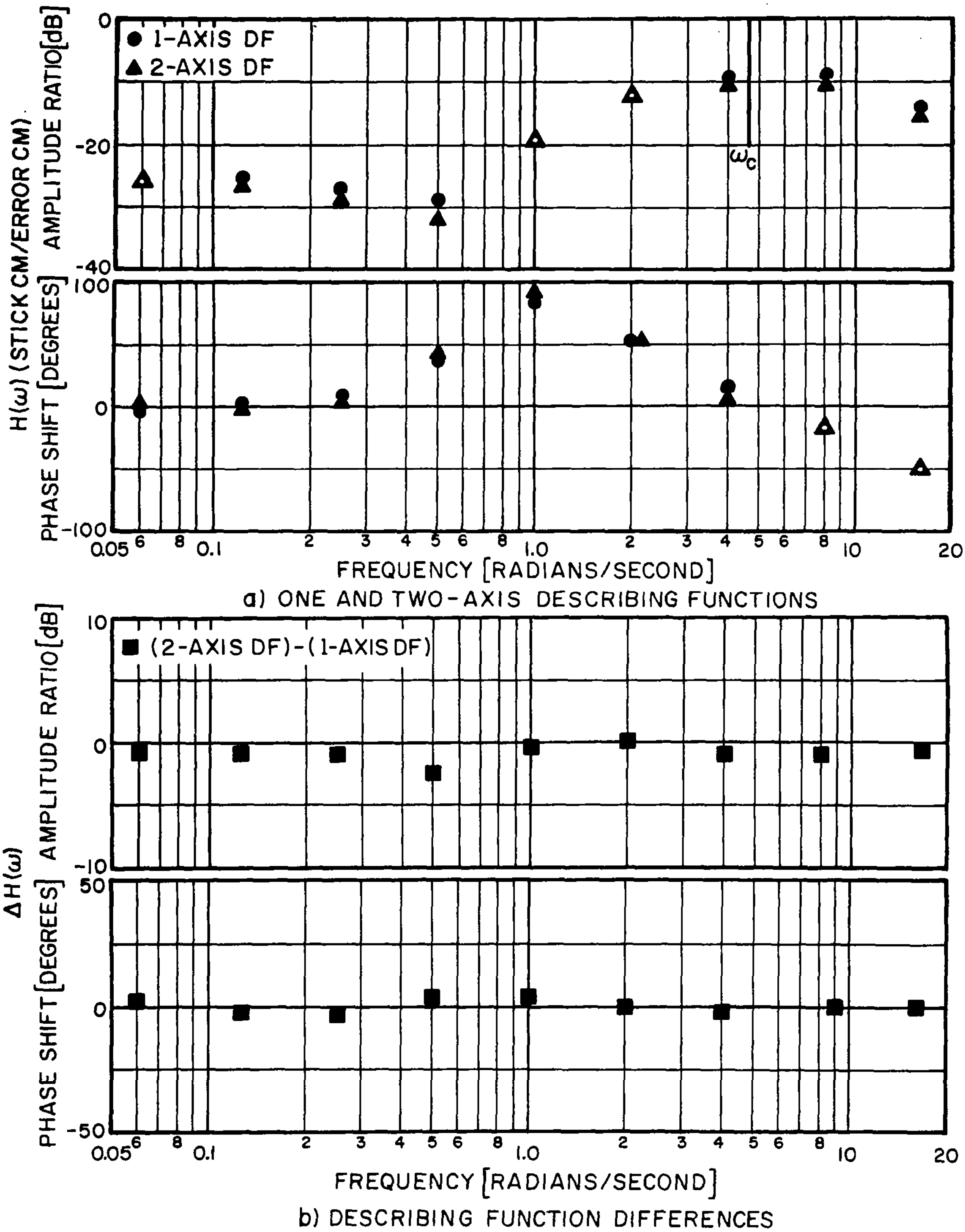


FIG. 18 HUMAN CONTROLLER DESCRIBING FUNCTIONS
Experiment 1 Homogeneous Control Situation
Input BW = 3.5 Rad/Sec, 3 Subj (3 Runs)

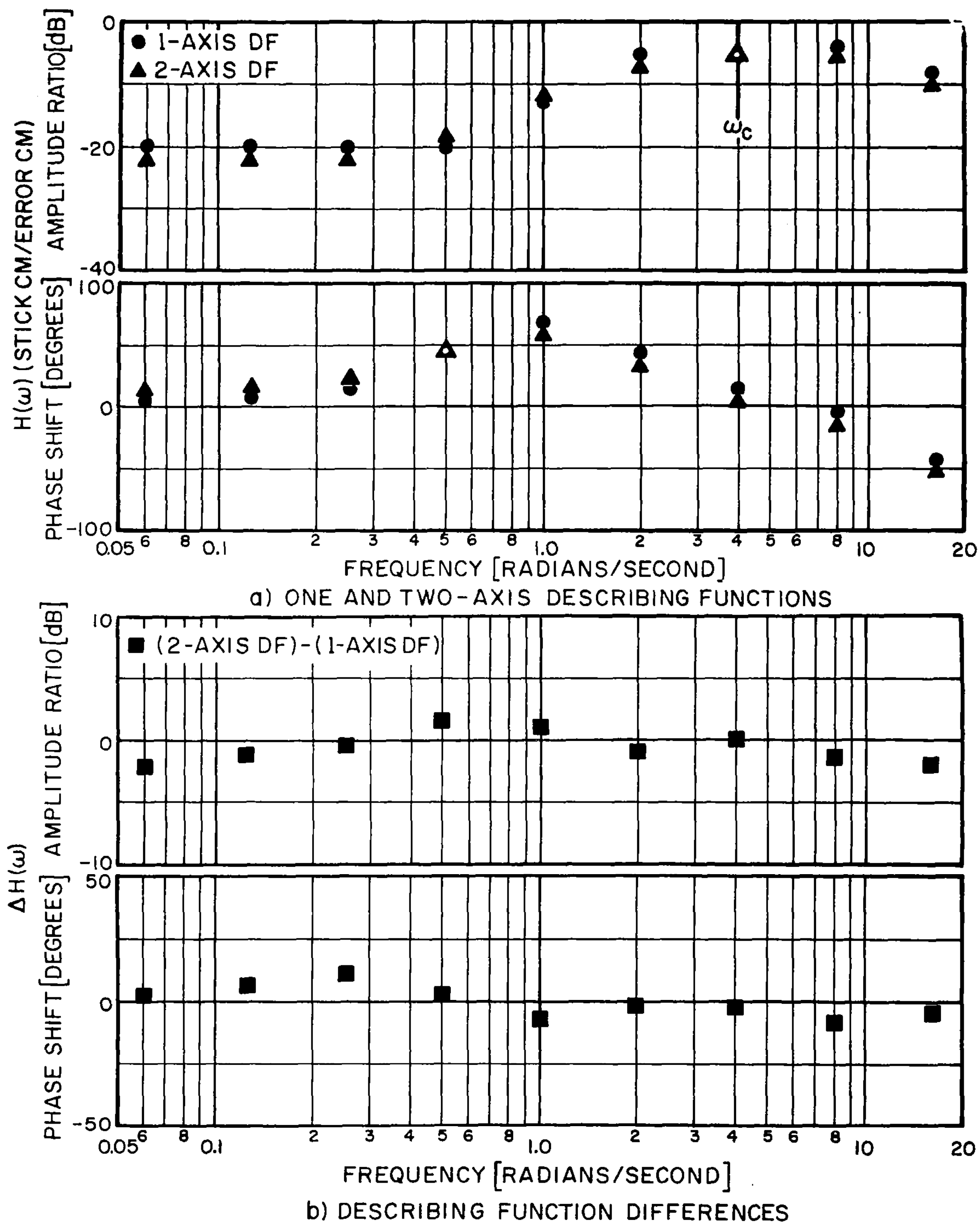


FIG. 19 HUMAN CONTROLLER DESCRIBING FUNCTIONS
Experiment 1 Homogeneous Control Situation
Input BW = 2.5 Rad/Sec, 3 Subj (3 Runs)

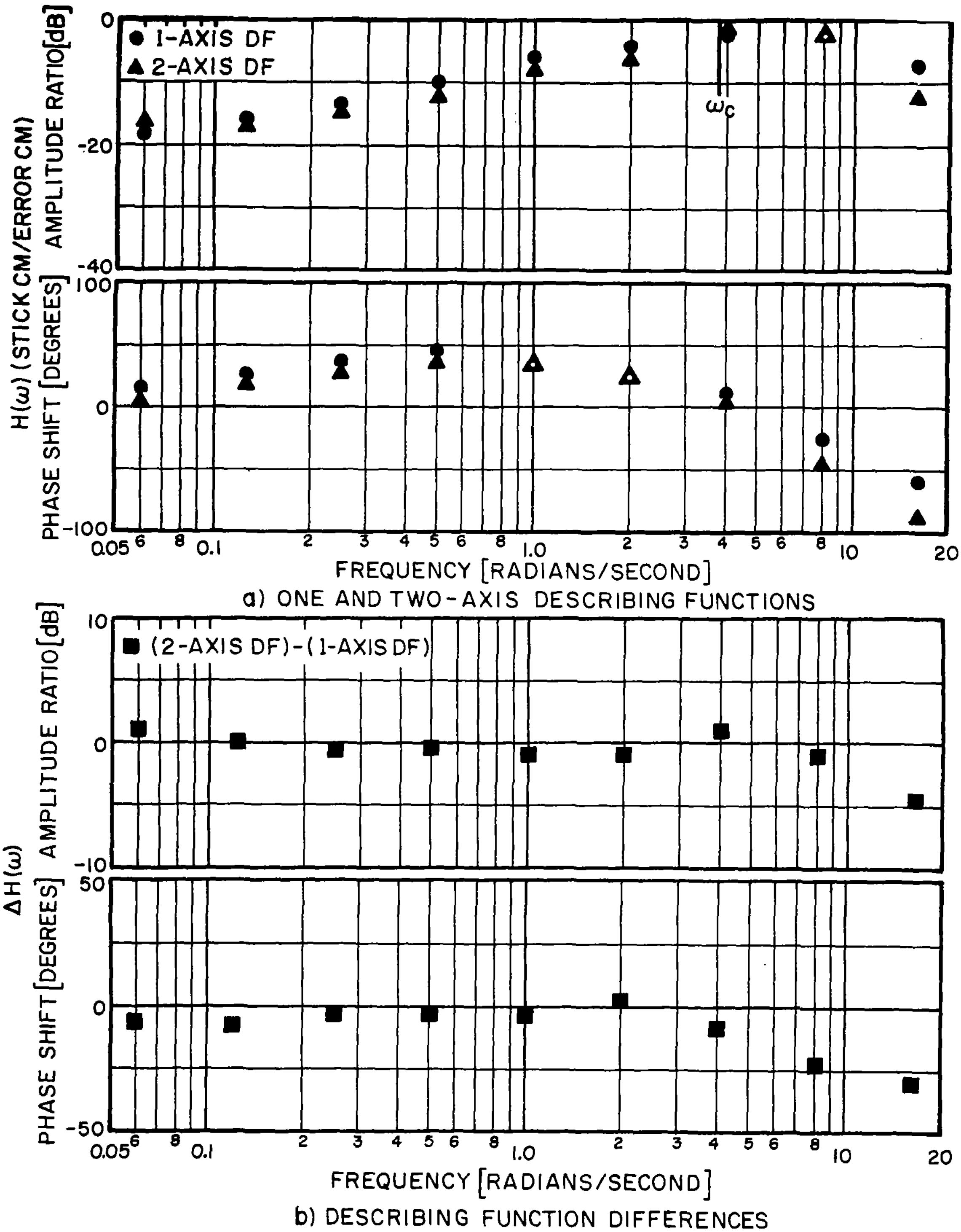


FIG. 20 HUMAN CONTROLLER DESCRIBING FUNCTIONS
Experiment 1 Homogeneous Control Situation
Input BW= 1.5 Rad/Sec, 3 Subj (3 Runs)

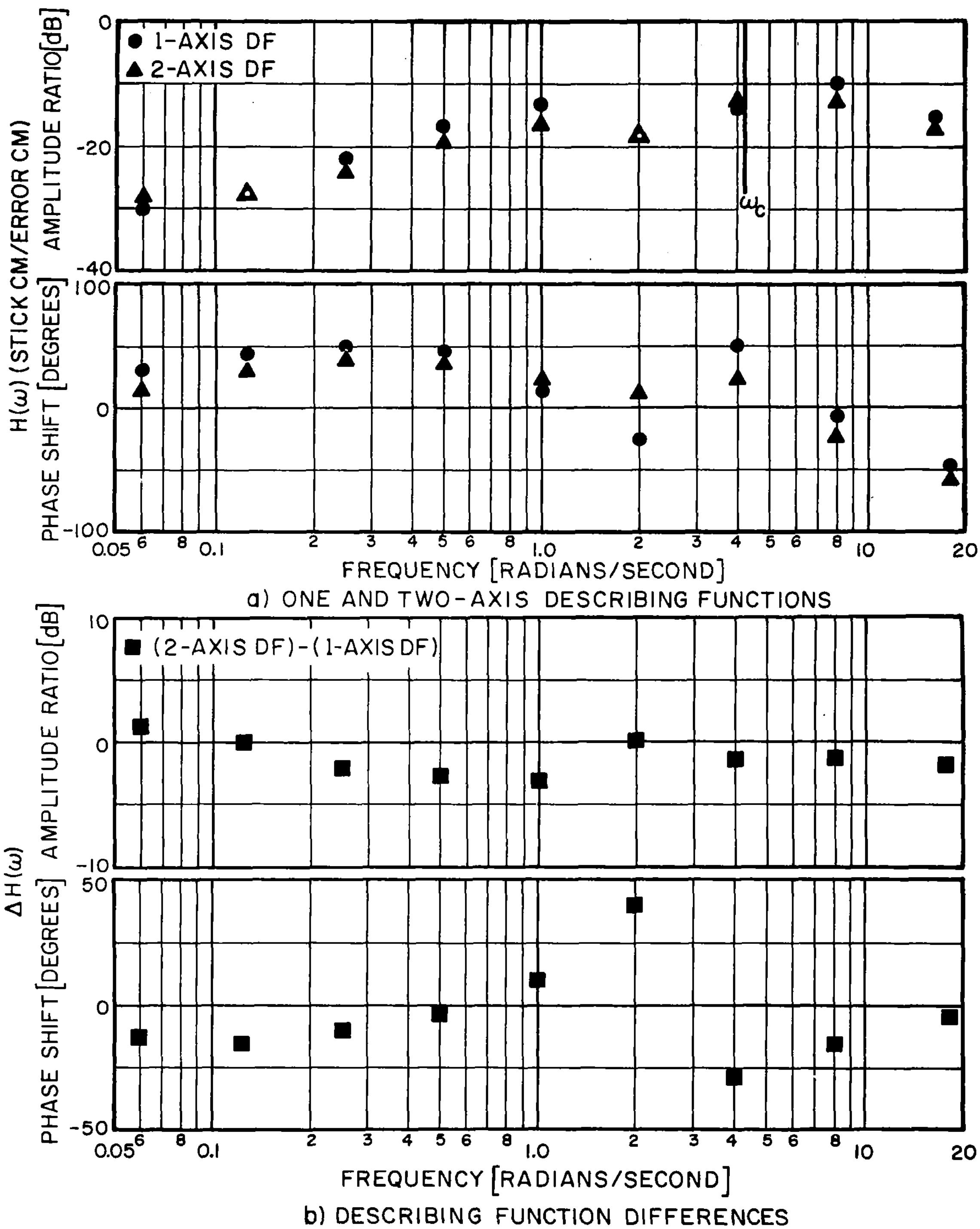


FIG. 21 HUMAN CONTROLLER DESCRIBING FUNCTIONS
Experiment 2 Heterogeneous Inputs
Input BW= 1.5 Rad/Sec, Subj RL (1 Run)

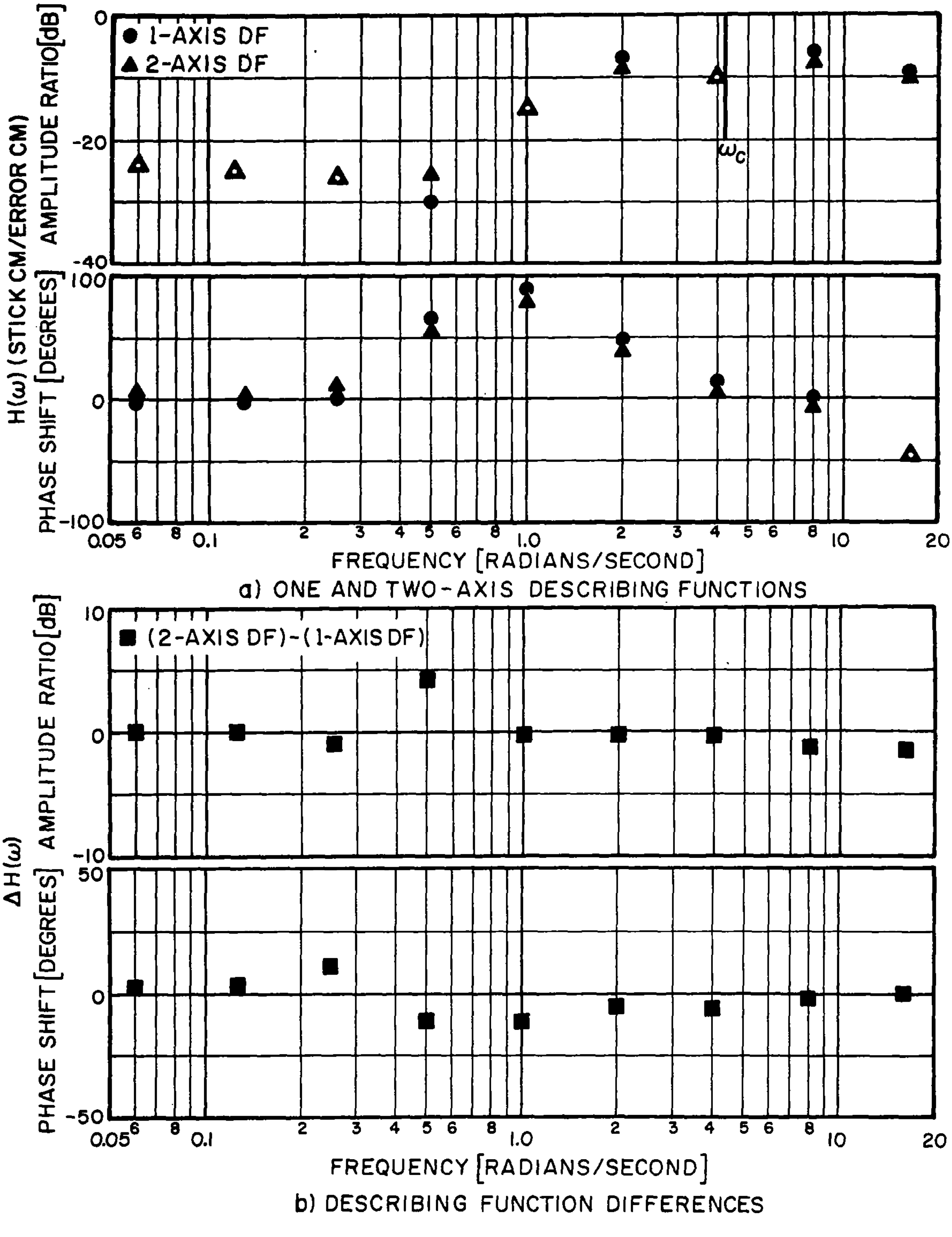


FIG. 22 HUMAN CONTROLLER DESCRIBING FUNCTIONS
Experiment 2 Heterogeneous Inputs
Input BW= 3.5 Rad/Sec, Subj RL (1 Run)

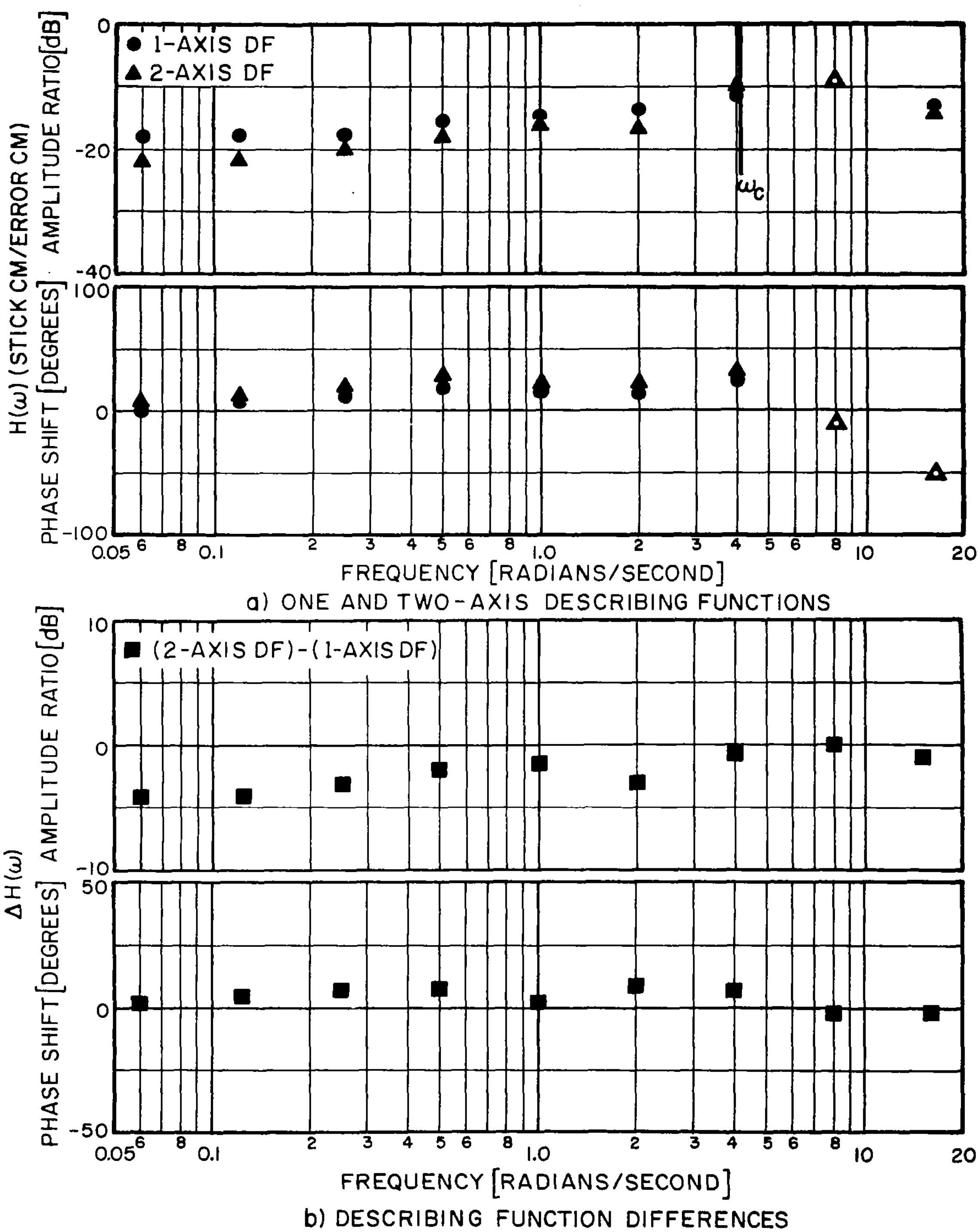
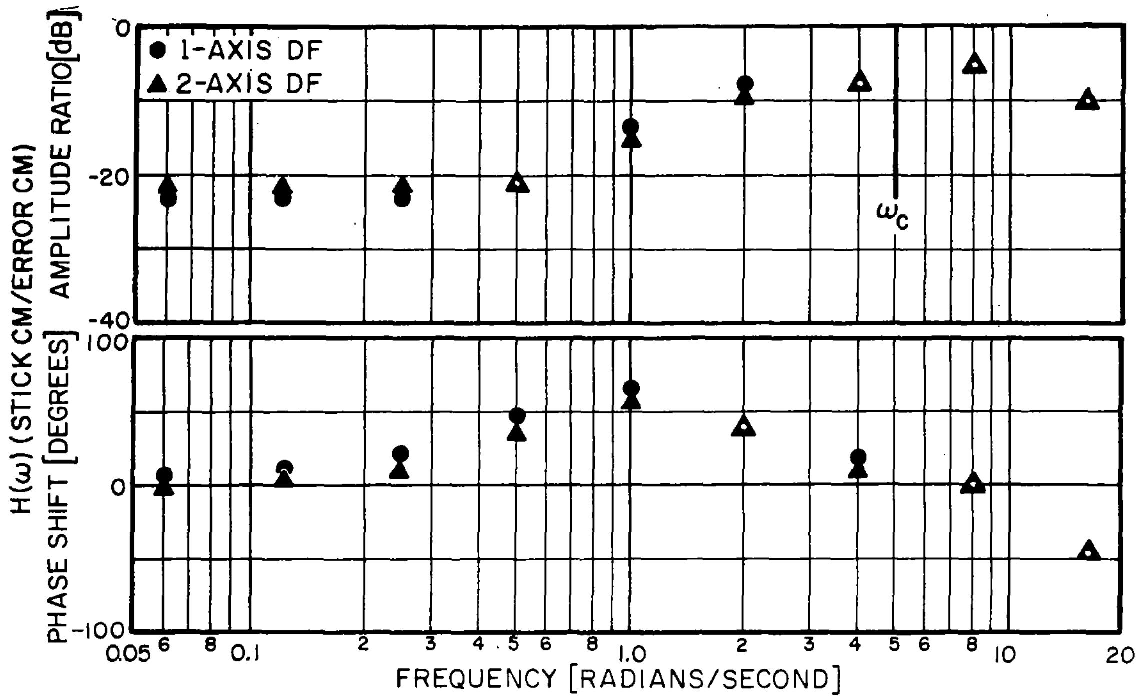
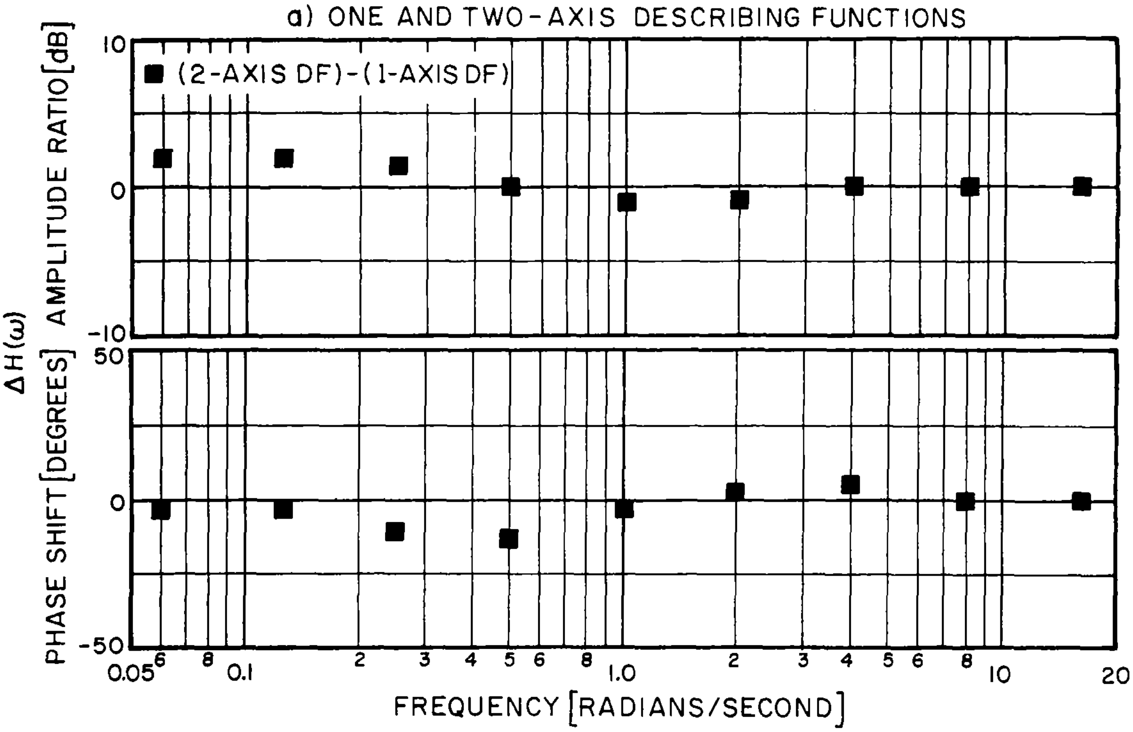


FIG. 23 HUMAN CONTROLLER DESCRIBING FUNCTIONS
Experiment 2 Heterogeneous Inputs
Input BW= 1.5 Rad/Sec, Subj CP (1 Run)

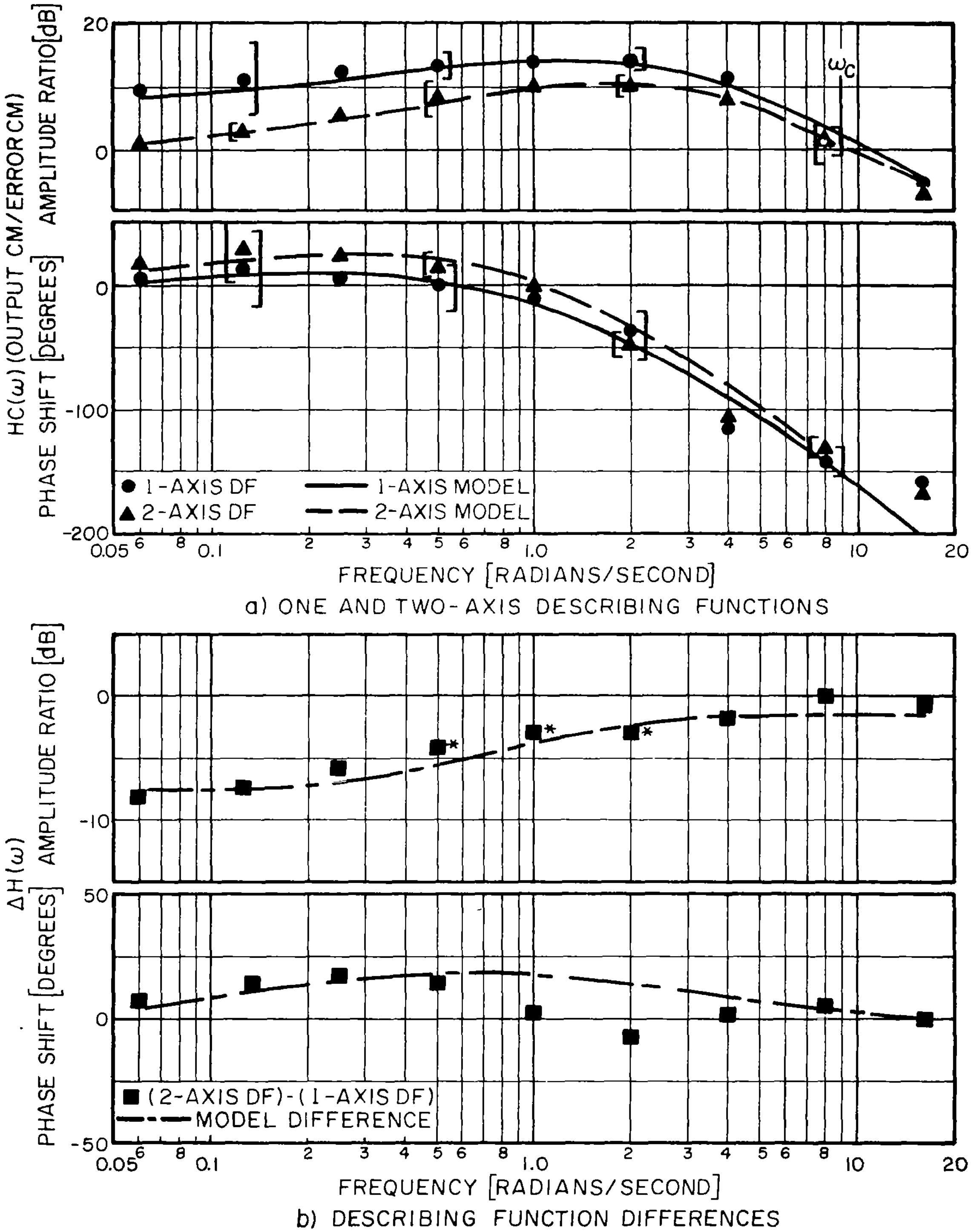


a) ONE AND TWO-AXIS DESCRIBING FUNCTIONS



b) DESCRIBING FUNCTION DIFFERENCES

FIG. 24 HUMAN CONTROLLER DESCRIBING FUNCTIONS
Experiment 2 Heterogeneous Inputs
Input BW= 3.5 Rad/Sec, Subj CP (1 Run)



'IG. 25 OPEN LOOP DESCRIBING FUNCTIONS
Experiment 3 Heterogeneous Dynamics
C=4, 3 Subj (9 Runs)

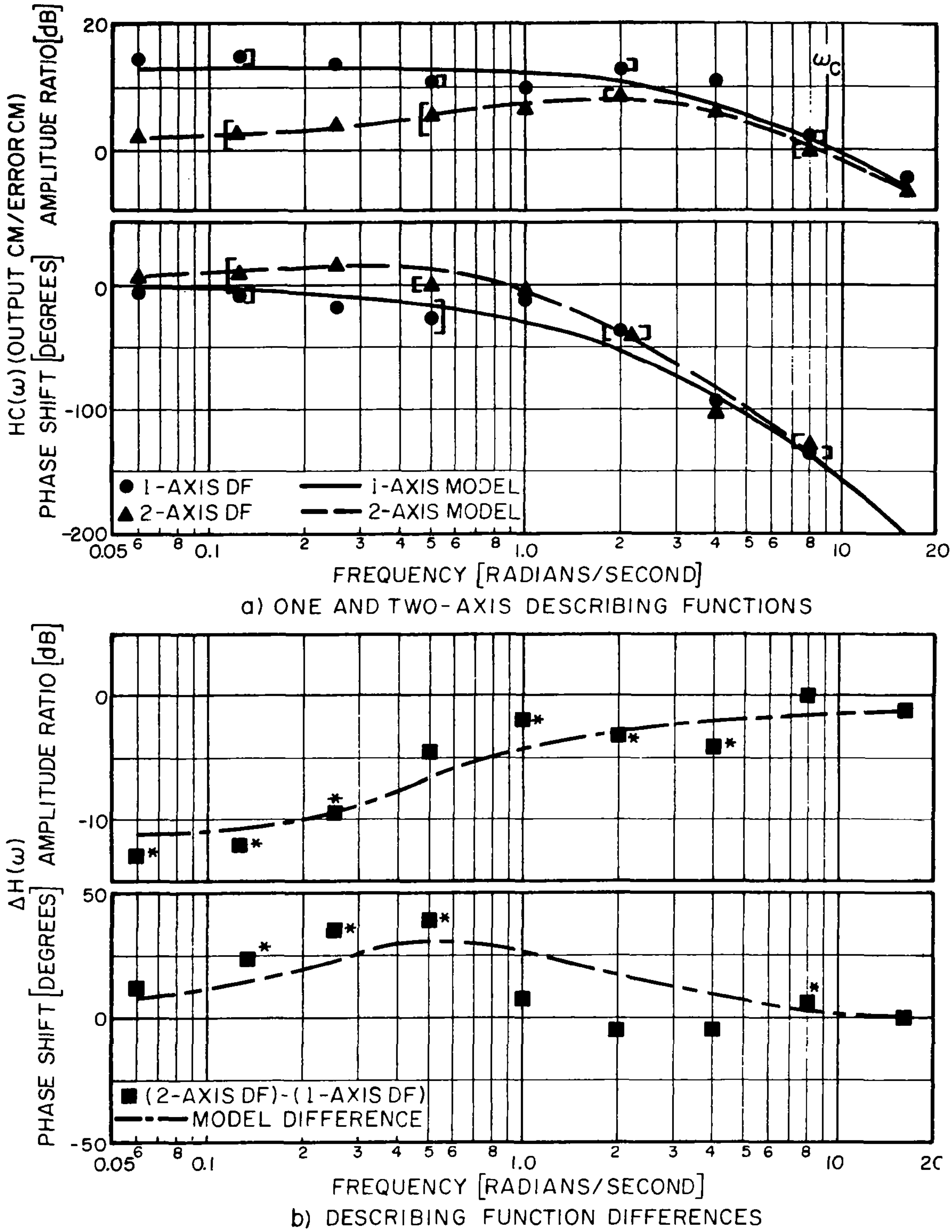


FIG. 26 OPEN LOOP DESCRIBING FUNCTIONS
Experiment 3 Heterogeneous Dynamics
C=4, Subj RL (3 Runs)

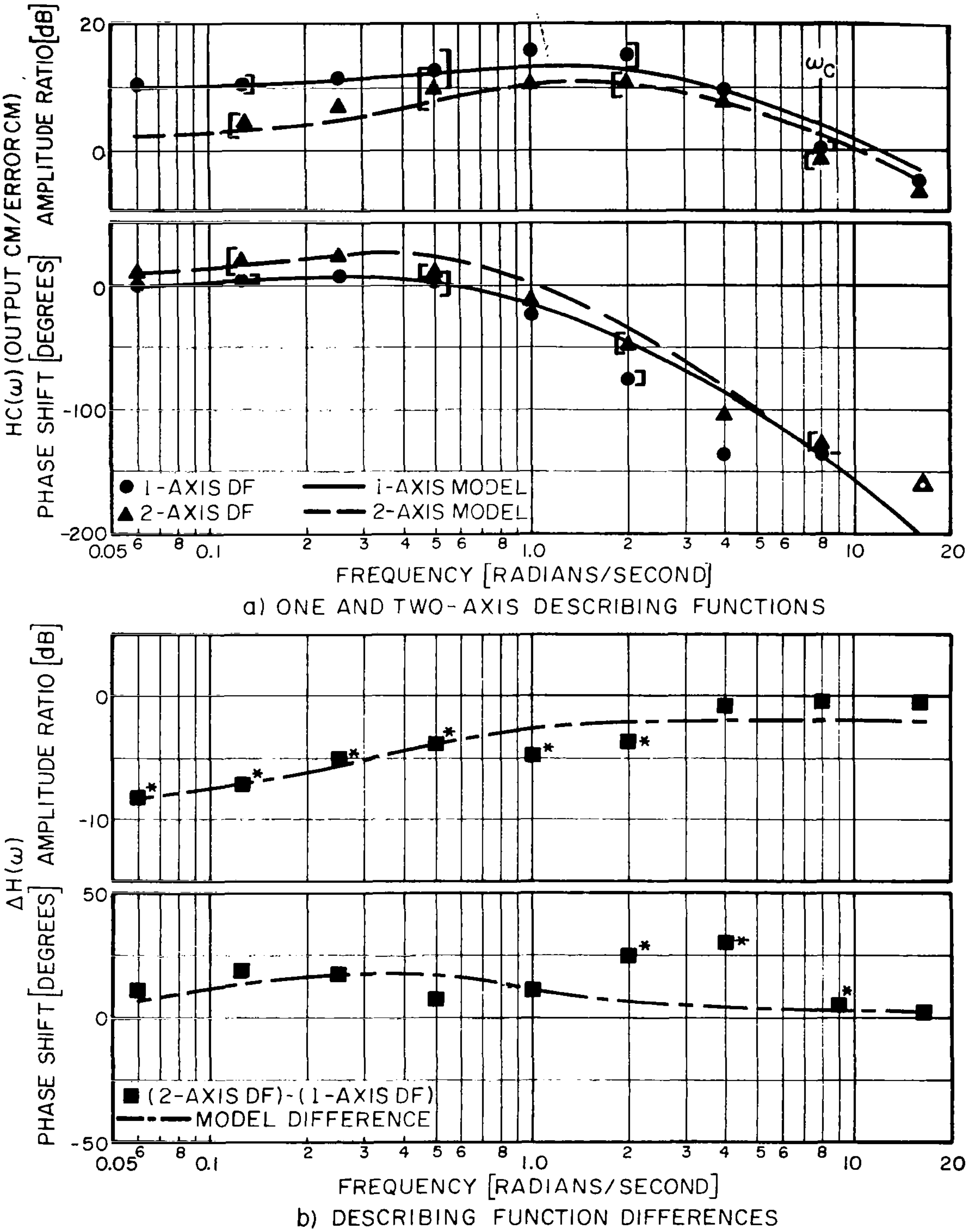


FIG. 27 OPEN LOOP DESCRIBING FUNCTIONS
Experiment 3 Heterogeneous Dynamics
C=4, Subj EK (3 Runs)

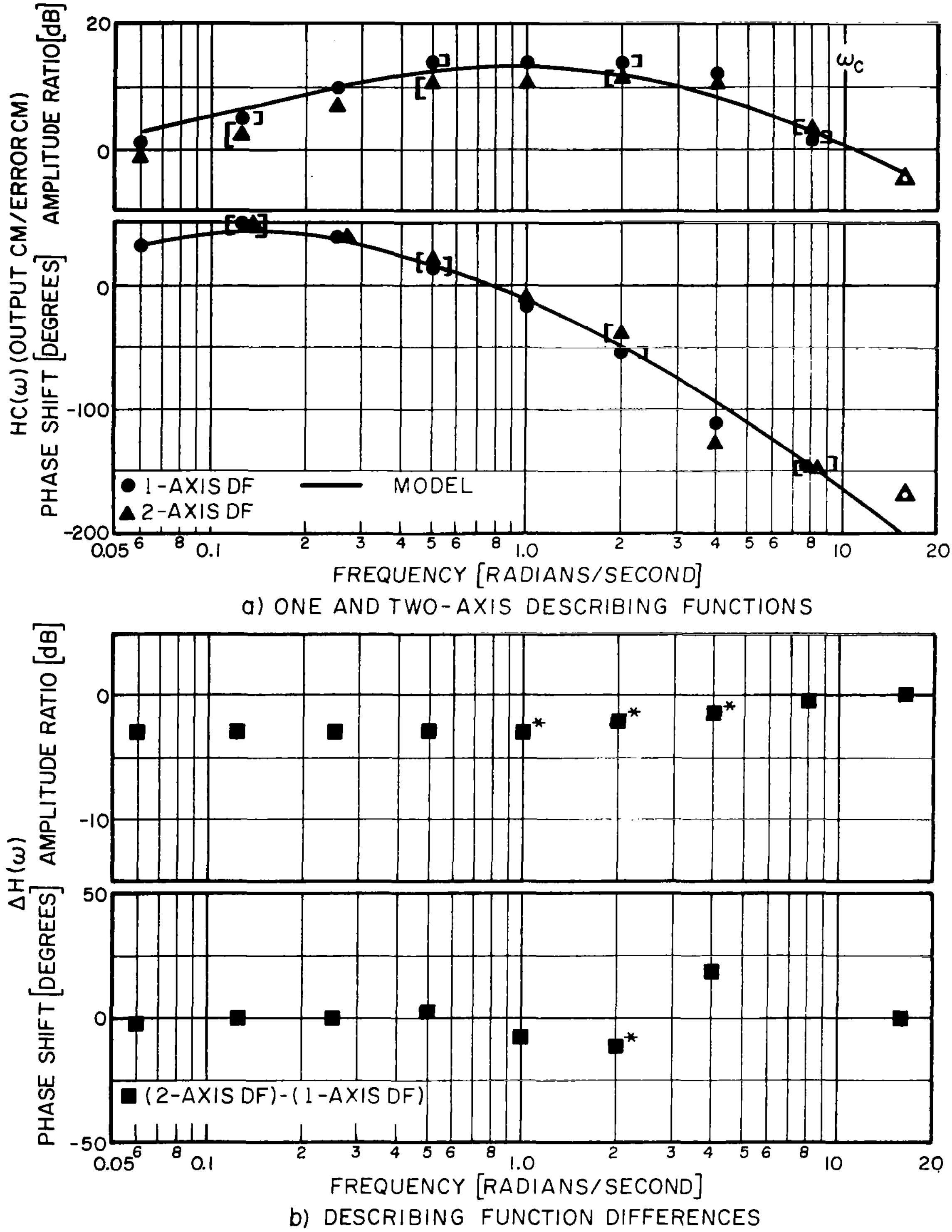


FIG. 28 OPEN LOOP DESCRIBING FUNCTIONS
Experiment 3 Heterogeneous Dynamics
C=4, Subj CP

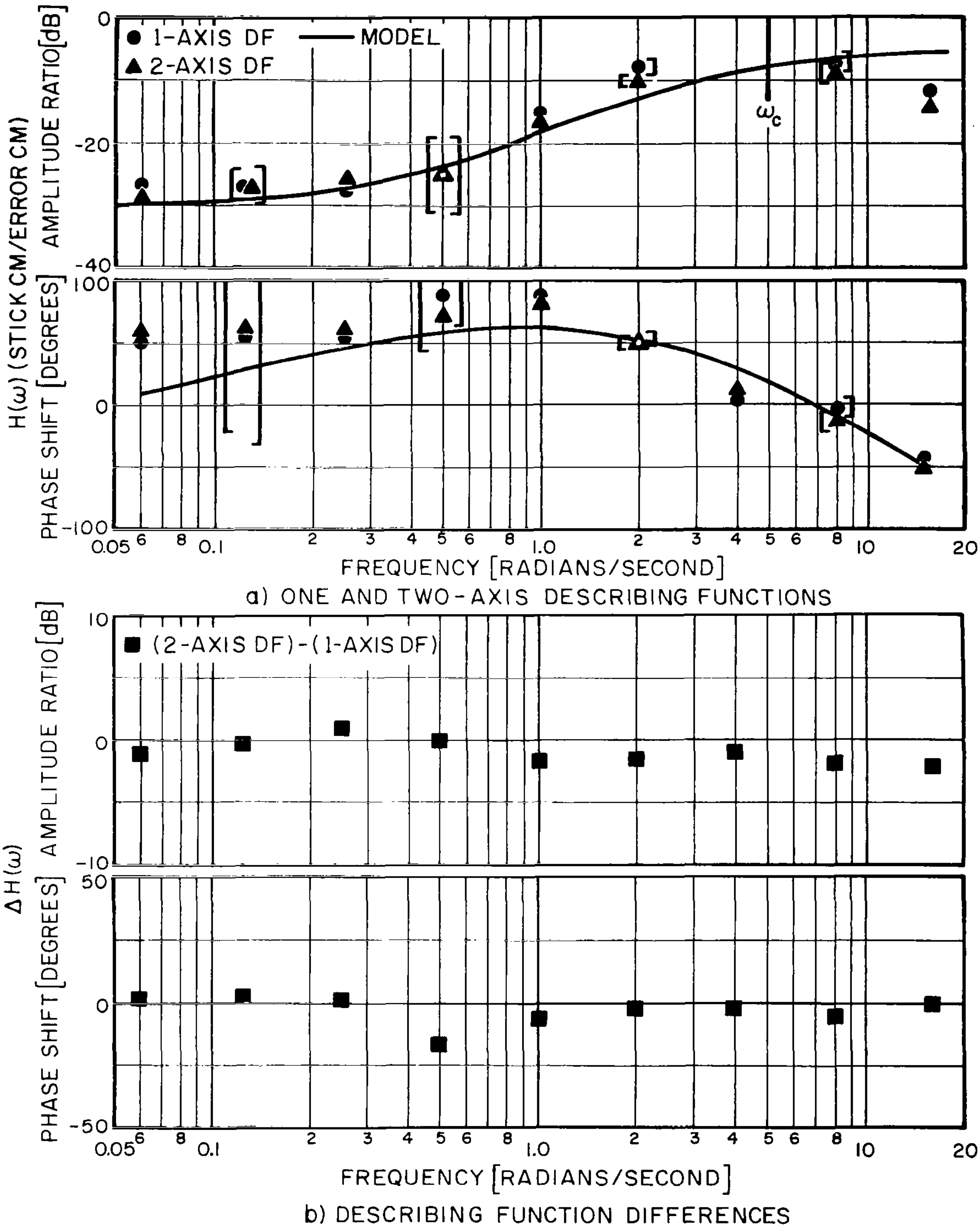


FIG. 29 HUMAN CONTROLLER DESCRIBING FUNCTIONS
Experiment 3 Heterogeneous Dynamics
C=64/s2, 3 Subj (9 Runs)

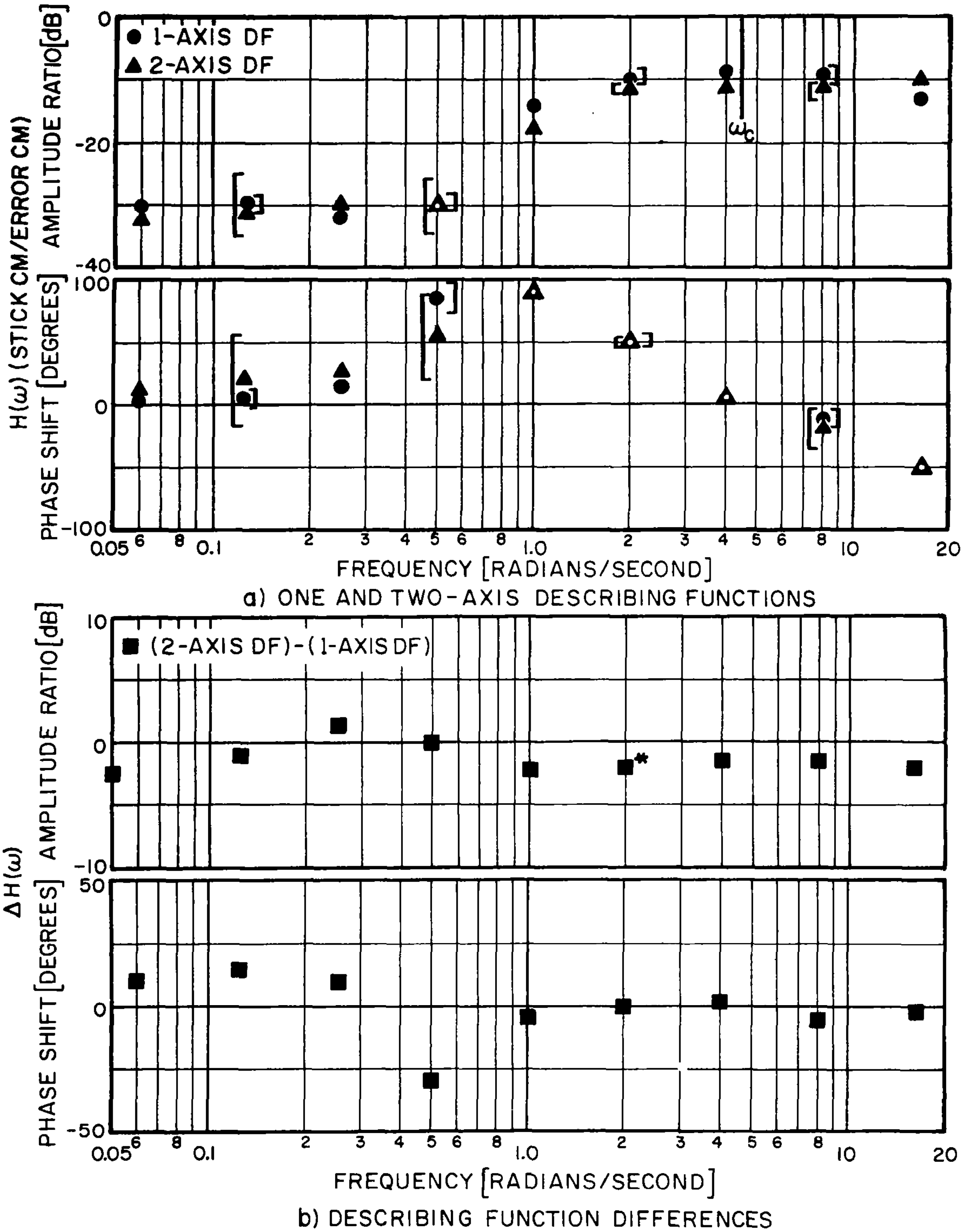


FIG. 30 HUMAN CONTROLLER DESCRIBING FUNCTIONS
Experiment 3 Heterogeneous Dynamics
C=64/s2, Subj RL (3 Runs)

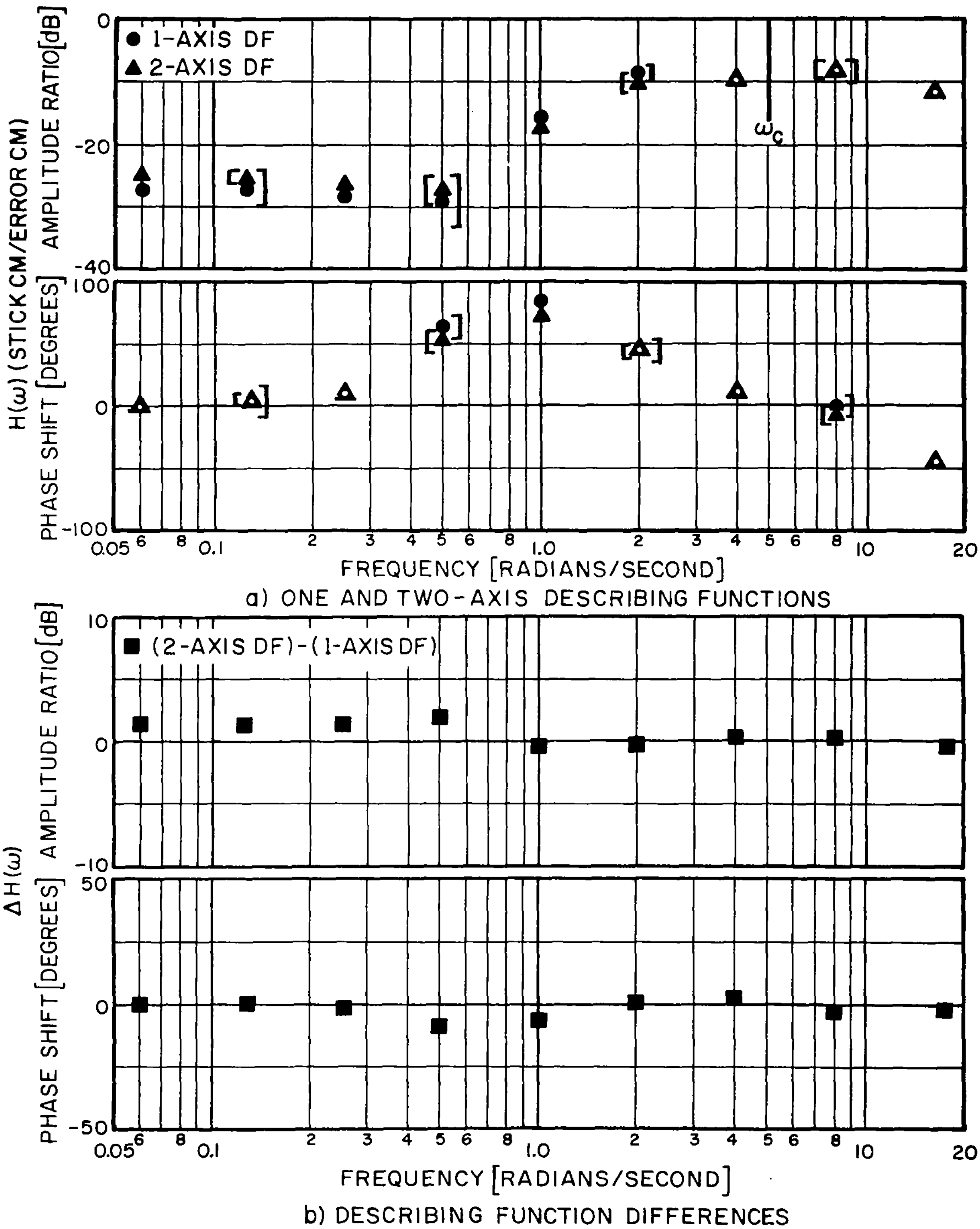


FIG. 31 HUMAN CONTROLLER DESCRIBING FUNCTIONS
Experiment 3 Heterogeneous Dynamics
C=64/s2, Subj EK (3 Runs)

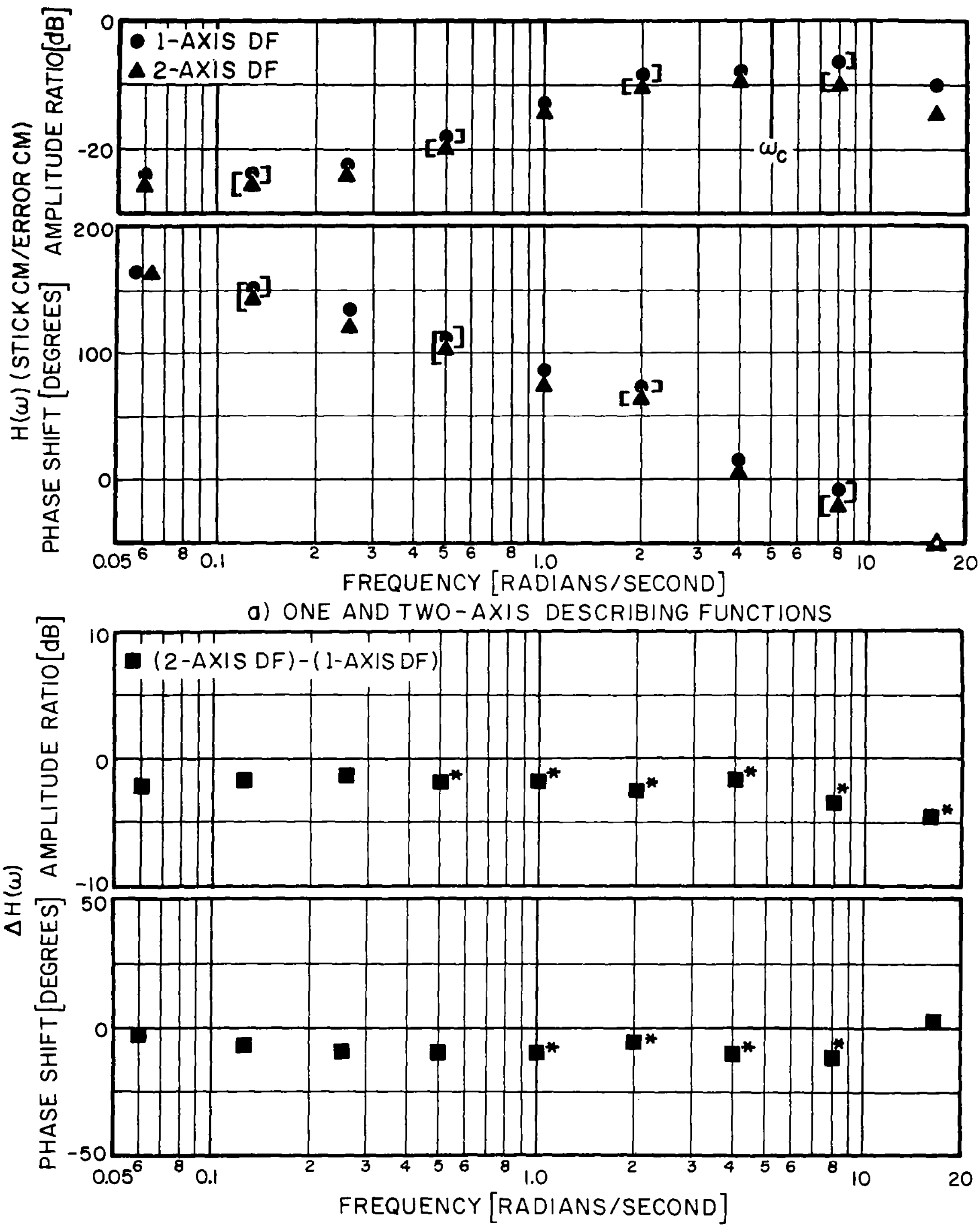


FIG. 32 HUMAN CONTROLLER DESCRIBING FUNCTIONS
Experiment 3 Heterogeneous Dynamics
C=64/s2, Subj CP (3 Runs)

APPENDIX A

PARAMETERS OF THE FILTERS USED IN MULTIPLE REGRESSION ANALYSIS

The pertinent control signals were converted to digital format at the rate of 10 samples per second. The parameters of the digital filters used in the computation of describing functions were chosen to be optimum for each measurement situation. The three measurement situations and the corresponding filter parameters are listed below.

1. Human controller describing function, $C(s) = K$

   Filter Poles: .5, 1, 2, 4, 8 radians per second
   Time Delay: 0.1 second.

2. Input-Output describing function, $C(s) = K$

   Filter Poles: 1, 2, 4, 8, 16 radians per second
   Time Delay: 0.1 second.

3. Human controller describing function, $C(s) = K/s^2$

   Filter Poles: 2, 2.8, 4, 5.7, 8 radians per second
   Time Delay: none (Sufficient phase lag was provided by the rational-polynomial portion of the filter to simulate the human controller's time delay.)

APPENDIX B
ANALYSES OF VARIANCE FOR THE NORMALIZED MEAN SQUARED ERROR

TABLE B1

Analysis of Variance for NMSE

Experiment 1: Homogeneous Control Situation
Input Bandwidth = 3.5 rad/sec

X axis

Source	Sums of Squares x 100	Degrees of Freedom	Mean Square x 100	F-Ratio
Within	5.5	36	0.15	
Number of Axes	0.08	1	0.08	<1.0
Signal Segment	5.2	2	2.6	17***
Subjects	0.37	2	0.19	1.3
Seg. x No.	0.65	2	0.33	2.2
Subj. x No.	0.70	2	0.35	2.3
Subj. x Seg.	1.1	4	0.27	1.8
No. x Seg. x Subj.	0.76	4	0.19	1.3

* Significant at the .05 level

** Significant at the .01 level

*** Significant at the .001 level

TABLE B2

Analysis of Variance for NMSE

Experiment 1: Homogeneous Control Situation
Input Bandwidth = 3.5 rad/sec

Y axis

Source	Sums of Squares x 100	Degrees of Freedom	Mean Square x 100	F-Ratio
Within	6.9	36	0.19	
Number of Axes	3.1	1	3.1	16***
Signal Segment	1.9	2	0.93	4.9**
Subjects	0.42	2	0.21	1.1
Seg. x No.	0.18	2	0.09	<1.0
Subj. x No.	0.38	2	0.19	1.0
Subj. x Seg.	1.6	4	0.39	2.1
No. x Seg. x Subj.	0.90	4	0.23	1.2

* Significant at the .05 level

** Significant at the .01 level

*** Significant at the .001 level

TABLE B3

Analysis of Variance for NMSE

Experiment 1: Homogeneous Control Situation
Input Bandwidth = 3.5 rad/sec

Total Task

Source	Sums of Squares x 100	Degrees of Freedom	Mean Square x 100	F-Ratio
Within	4.1	36	0.11	
Number of Axes	1.1	1	1.1	10**
Signal Segment	3.3	2	1.6	15***
Subjects	0.02	2	0.01	<1.0
Seg. x No.	0.33	2	0.17	1.5
Subj. x No.	0.37	2	0.18	1.6
Subj. x Seg.	0.98	4	0.25	2.3
No. x Seg. x Subj.	0.76	4	0.19	1.7

* Significant at the .05 level

** Significant at the .01 level

*** Significant at the .001 level

TABLE B4

Analysis of Variance for NMSE

Experiment 1: Homogeneous Control Situation
Input Bandwidth = 2.5 rad/sec

X axis

Source	Sums of Squares x 400	Degrees of Freedom	Mean Square x 400	F-Ratio
Within	8.2	36	0.23	
Number of Axes	0.26	1	0.26	1.1
Signal Segment	1.1	2	0.56	2.4
Subjects	12	2	6.0	26***
Seg. x No.	0.13	2	0.06	<1.0
Subj. x No.	0.18	2	0.09	<1.0
Subj. x Seg.	0.12	4	0.03	<1.0
No. x Seg. x Subj.	0.11	4	0.03	<1.0

* Significant at the .05 level

** Significant at the .01 level

*** Significant at the .001 level

TABLE B5

Analysis of Variance for NMSE

Experiment 1: Homogeneous Control Situation
Input Bandwidth = 2.5 rad/sec

Y axis

Source	Sums of Squares x 400	Degrees of Freedom	Mean Square x 400	F-Ratio
Within	3.2	36	0.088	
Number of Axes	1.6	1	1.6	4.6
Signal Segment	4.8	2	2.4	27***
Subjects	3.2	2	1.6	<1.0
Seg. x No.	0.00	2	0.00	<1.0
Subj. x No.	0.70	2	0.35	4.0*
Subj. x Seg.	0.63	4	0.16	1.8
No. x Seg. x Subj.	0.64	4	0.16	1.8

* Significant at the .05 level

** Significant at the .01 level

*** Significant at the .001 level

TABLE B6

Analysis of Variance for NMSE

Experiment 1: Homogeneous Control Situation
Input Bandwidth = 2.5 rad/sec

Total Task

Source	Sums of Squares x 400	Degrees of Freedom	Mean Square x 400	F-Ratio
Within	3.6	36	0.10	
Number of Axes	0.14	1	0.14	1.4
Signal Segment	0.97	2	0.48	4.8*
Subjects	6.9	2	3.4	34***
Seg. x No.	0.04	2	0.02	<1.0
Subj. x No.	0.05	2	0.02	<1.0
Subj. x Seg.	0.24	4	0.06	<1.0
No. x Seg. x Subj.	0.28	4	0.07	<1.0

* Significant at the .05 level

** Significant at the .01 level

*** Significant at the .001 level

TABLE B7

Analysis of Variance for NMSE

Experiment 1: Homogeneous Control Situation
Input Bandwidth = 1.5 rad/sec

X axis

Source	Sums of Squares $\times 10^4$	Degrees of Freedom	Mean Square $\times 10^4$	F-Ratio
Within	20	36	0.55	
Number of Axes	0.02	1	0.02	<1.0
Signal Segment	5.9	2	3.0	5.5**
Subjects	92	2	46	84***
Seg. x No.	0.85	2	0.42	<1.0
Subj. x No.	2.8	2	1.4	2.5
Subj. x Seg.	5.7	4	1.4	2.5
No. x Seg. x Subj.	0.44	4	0.11	<1.0

* Significant at the .05 level

** Significant at the .01 level

*** Significant at the .001 level

TABLE B8

Analysis of Variance for NMSE

Experiment 1: Homogeneous Control Situation
Input Bandwidth = 1.5 rad/sec

Y axis

Source	Sums of Squares x 10^4	Degrees of Freedom	Mean Square x 10^4	F-Ratio
Within	17	36	0.47	
Number of Axes	9.4	1	9.4	20***
Signal Segment	2.6	2	1.3	2.7
Subjects	7.8	2	3.9	8.3**
Seg. x No.	1.8	2	0.91	1.9
Subj. x No.	2.0	2	1.0	2.1
Subj. x Seg.	0.86	4	0.22	<1.0
No. x Seg. x Subj.	0.47	4	0.12	<1.0

* Significant at the .05 level

** Significant at the .01 level

*** Significant at the .001 level

TABLE B9

Analysis of Variance for NMSE

Experiment 1: Homogeneous Control Situation
Input Bandwidth = 1.5 rad/sec

Total Task

Source	Sums of Squares x 400	Degrees of Freedom	Mean Square x 400	F-Ratio
Within	11	36	0.30	
Number of Axes	1.9	1	1.9	6.3*
Signal Segment	4.2	2	2.1	7.0
Subjects	39	2	19	20**
Seg. x No.	0.32	2	0.16	<1.0
Subj. x No.	1.6	2	0.79	2.6
Subj. x Seg.	3.9	4	0.97	3.2
No. x Seg. x Subj.	0.38	4	0.09	<1.0

* Significant at the .05 level

** Significant at the .01 level

*** Significant at the .001 level

TABLE B10

Analysis of Variance for NMSE

Experiment 2: Heterogeneous Inputs, Homogeneous Dynamics

Input Bandwidth = 1.5 rad/sec

X axis

Source	Sums of Squares $\times 10^4$	Degrees of Freedom	Mean Square $\times 10^4$	F-Ratio
Within	5.2	24	0.22	
Number of Axes	8.5	1	8.5	2.0
Signal Segment	2.1	2	1.0	<1.0
Subjects	3.7	1	3.7	<1.0
Seg. x No.	0.74	2	0.37	1.7
Subj. x No.	4.2	1	4.2	20***
Subj. x Seg.	2.4	2	1.2	5.5**
No. x Seg. x Subj.	0.66	2	0.33	1.5

* Significant at the .05 level

** Significant at the .01 level

*** Significant at the .001 level

TABLE B11

Analysis of Variance for NMSE

Experiment 2: Heterogeneous Inputs, Homogeneous Dynamics
Input Bandwidth = 3.5 rad/sec

Y axis

Source	Sums of Squares x 100	Degrees of Freedom	Mean Square x 100	F-Ratio
Within	0.76	24	0.03	
Number of Axes	0.06	1	0.06	2.0
Signal Segment	0.07	2	0.03	<1.0
Subjects	0.48	1	0.48	<1.0
Seg. x No.	0.09	2	0.05	1.7
Subj. x No.	0.03	1	0.03	1.0
Subj. x Seg.	0.27	2	0.13	4.2*
No. x Seg. x Subj.	0.03	2	0.02	<1.0

* Significant at the .05 level

** Significant at the .01 level

*** Significant at the .001 level

TABLE B12

Analysis of Variance for NMSE

Experiment 2: Heterogeneous Inputs, Homogeneous Dynamics

Total Task

Source	Sums of Squares x 10^4	Degrees of Freedom	Mean Square x 10^4	F-Ratio
Within	17	24	0.71	
Number of Axes	0.08	1	0.08	<1.0
Signal Segment	0.17	2	0.09	<1.0
Subjects	20	1	20	28***
Seg. x No.	0.61	2	0.31	<1.0
Subj. x No.	0.44	1	0.44	<1.0
Subj. x Seg.	4.0	2	2.0	2.8
No. x Seg. x Subj.	1.4	2	0.71	1.0

* Significant at the .05 level

** Significant at the .01 level

*** Significant at the .001 level

TABLE B13

Analysis of Variance for NMSE

Experiment 3: Heterogeneous Dynamics, Homogeneous Inputs

K axis

Source	Sums of Squares $\times 10^4$	Degrees of Freedom	Mean Square $\times 10^4$	F-Ratio
Within	52	48	1.1	
Number of Axes	228	1	228	19*
Subjects	62	2	31	2.0
Subj. x No.	24	2	12	11***

* Significant at the .05 level

** Significant at the .01 level

*** Significant at the .001 level

TABLE B14

Analysis of Variance for NMSE

Experiment 3: Heterogeneous Dynamics, Homogeneous Inputs

K/s^2 axis

Source	Sums of Squares x 100	Degrees of Freedom	Mean Square x 100	F-Ratio
Within	6.0	48	0.13	
Number of Axes	8.1	1	8.1	6.7
Subjects	3.6	2	1.8	1.5
Subj. x No.	2.5	2	1.2	9.2***

* Significant at the .05 level

** Significant at the .01 level

*** Significant at the .001 level

TABLE B15

Analysis of Variance for NMSE

Experiment 3: Heterogeneous Dynamics, Homogeneous Inputs

Total Task

Source	Sums of Squares x 400	Degrees of Freedom	Mean Square x 400	F-Ratio
Within	5.0	48	0.10	
Number of Axes	18	1	18	16
Subjects	5.4	2	2.7	2.5
Subj. x No.	2.2	2	1.1	11***

* Significant at the .05 level

** Significant at the .01 level

*** Significant at the .001 level

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